Are We There Yet?
Mechanism Design Beyond Equilibrium

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Abstract

There are mechanism design problems for which an exclusive focus on equilibrium can be seriously misleading. If outcomes will be implemented whether or not an equilibrium has been achieved, then the desiderata by which we evaluate mechanisms in these situations need to include more than merely the properties of their equilibria (are the equilibria Pareto optimal; are they in dominant strategies; are they stable; etc.). For the classical public-goods problem, we describe some of our research in which (1) we showed, in an experiment, that several mechanisms with excellent equilibrium properties exhibited serious out-of-equilibrium failures; (2) by emulating the Walrasian exchange model, we designed a public-good mechanism to be transparent and to have reasonable properties even when out of equilibrium; and (3) we conducted an experiment in which this new mechanism performed better than previous mechanisms.

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In mechanism design, Leo Hurwicz created a new field of economics. He used the theory to bring informational and incentive issues to the fore, and to address two fundamental questions in economics: in classical allocation problems, what can economic institutions achieve, and what can't they achieve? (Hurwicz 1979, 1990). The tool he used, and the one we've all used ever since, is game theoretic equilibrium.

The two classical allocation problems that Leo addressed were price formation in the pure exchange problem and the "free rider problem" with public goods. In both problems it's noteworthy that we often implement outcomes in real time, as our institutions produce them, rather than waiting to attain an equilibrium and asking along the way "Are we there yet?"

Realistically, we are probably never really at an equilibrium. Equilibrium predictions are useful when we think we will at least be "close to" an equilibrium, reasonably quickly, or when our interest is primarily in a system's long-run state.

But if outcomes are going to be implemented in or out of equilibrium, then clearly we need to know something about disequilibrium outcomes. Knowing only about equilibrium outcomes is not good enough.

We view this as a variation on Wilson's argument for "robust" mechanism design (Wilson 1987). Wilson's emphasis was on the theory's assumption of common knowledge of the participants' preferences and information. Milgrom subsequently went further, maintaining that "the behavior of [a mechanism's participants] cannot be regarded as perfectly predictable" (Milgrom 2004). Milgrom's larger point was that when mechanism design is required to actually perform well on the ground, "mechanisms that are optimized to perform well when the assumptions are exactly true may still fail miserably in the much more frequent cases when the assumptions are untrue."

We take this view a step further, treating the idea that the players in a game will play equilibrium strategies as an additional assumption. In some cases the equilibrium assumption is fruitful: the equilibrium prediction is close enough to the choices the players actually make, and this tells us with some degree of accuracy what the welfare implications will be and what will happen if we change the rules of the game or if some features of the environment change. But this requires either that we determine, for any given mechanism, whether actual participants will play "close enough" to equilibrium, or else that we design the mechanism in the first place to be "robust" to non-equilibrium play.

This is the approach we have taken in some recent research we describe here on public-good mechanisms. In a paper with Lazzati, we conducted an experiment with three well-known Lindahl mechanisms, mechanisms whose equilibria produce Lindahl allocations and prices. Over the course of many plays, by many subjects, equilibrium play was never observed in the experiment. Worse, the play that *was* observed often produced infeasible outcomes and outcomes with welfare properties that were much worse than the welfare properties of equilibrium outcomes. There are reasons to believe that the performance of these three mechanisms would be at least roughly representative of other public-goods mechanisms that economists have proposed. In order to provide the necessary incentives, the mechanisms use unintuitive outcome functions that do not seem to lead participants to the mechanisms' equilibria. Moreover, the features that provide these incentives also have the potential to create infeasible and otherwise undesirable outcomes when not in equilibrium.

These experimental results led us to devise a mechanism that would be intuitive, and if not necessarily optimal, at least satisfactory, whether in or out of equilibrium. The approach we adopted was to emulate simple mechanisms for attaining a Walrasian equilibrium, such as the mechanism introduced by Dubey (1982). The motivation here was that when there are only two persons, the problem of selecting an amount of a public good, together with the allocation of its cost, is exactly equivalent to the standard Edgeworth box two-person two-good exchange economy. (To see this geometrically, compare the Edgeworth box and the Kolm triangle.) Mechanisms such as Dubey's are transparent, relying on price and quantity proposals, so participants might be expected to play an equilibrium, or at least close to an equilibrium; the mechanisms also have desirable equilibria; and their outcomes are well-defined when not in equilibrium. Following this idea, we first defined a price-quantity mechanism for the two-person Edgeworth box problem, then reinterpreted it for the two-person public-good problem, and then generalized the public-good version of the mechanism to an arbitrary number of participants.

Does the new price-quantity mechanism perform any better than existing public-good mechanisms? Or, since it has many equilibria, most of which are not Pareto optimal, does it actually perform worse? We devised and conducted an experiment to answer that question.

We begin with a brief description of our experiment with three Lindahl mechanisms. We follow that with a description of the price-quantity mechanism. And we follow that in turn with a description of our experiment using the new price-quantity mechanism, and a comparison of its performance to the performance of the three Lindahl mechanisms.

An Experiment: Three Lindahl Mechanisms

In Van Essen, Lazzati, and Walker (VLW, 2012) we conducted an experiment to evaluate the performance of three mechanisms designed to achieve Lindahl allocations at their equilibria, the mechanisms introduced by Walker (1981), Kim (1993), and Chen (2002). The mechanisms were applied to the following simple public-good allocation problem: three participants must choose the quantity q of a public good and also how to allocate among themselves the cost of providing the q units.

Each of the three mechanisms requires each participant to choose an action, or message, m_i, and produces an outcome $(q, t_1, t_2, t_3) \in \mathbb{R}_+ \times \mathbb{R}^3$, where t_i denotes the tax to be paid by participant i; if $t_i < 0$, then i is *paid* $|t_i|$ dollars. The particular mechanism is defined by the domain from which participants may choose their messages m_i and by the outcome function φ that maps message profiles (m_1, m_2, m_3) into outcomes (q, t_1, t_2, t_3).

The subjects in the experiment were divided into groups of three and each three-person group used one of the three mechanisms repeatedly, forty times, to determine an outcome (q, t_1, t_2, t_3) in each of the 40 periods. Each group used the same mechanism for all 40 periods; each subject played the same role, $i = 1, 2$, or 3, at each of the 40 periods; and the subjects were paid for all 40 outcomes at the end of the experimental session.

The public good cost the group twelve experimental dollars (E$) per unit. Each group member $i = 1, 2, 3$ received a benefit of $v_i(q) = a_i q - q^2$ E$ when q units were provided, where $a_1 = 22, a_2 = 16, a_3 = 28$. The Pareto allocations are the ones that maximize the economic surplus $S(q) = \sum_1^3 v_i(q) - 12q$, *viz.* $\widehat{q} = 9$ and $S(\widehat{q}) = 243$. The Lindahl outcome is unique and independent of the mechanism: the Lindahl quantity is $q = 9$, the unique Pareto public good level, and the Lindahl taxes are $t_1 = 36, t_2 = -18, t_3 = 90$.

A total of 81 subjects participated in the experiment: nine three-subject groups for each mechanism. This provided, for each mechanism, 360 "plays" (9 groups times 40 periods) and 1080 individual decisions and outcomes (3 times 360). Altogether, for the three mechanisms, there were 1080 plays and 3240 individual decisions and outcomes.

We provide a brief summary of the results:

Equilibrium, Lindahl, and Pareto: In 1080 plays, Nash equilibrium and the Lindahl outcome were never observed; all outcomes were disequilibrium outcomes. The frequency of Pareto outcomes ($q = 9$ and $S(q) = 243$) is described in Table 1: 25 plays in the Chen mechanism (7% of the 360 plays); 19 plays in the Kim mechanism (5%); and 13 plays in the Walker mechanism (3.6%).

Economic surplus: Figure 1 and Table 1 describe the distributions of the economic surplus $S(q)$ earned by the groups: the Kim and Chen mechanisms produced very similar means, much larger than the Walker mechanism's mean; and the Kim mechanism produced much less variability across groups than the other two mechanisms.

Table 1

	Mean	Std Dev
Kim	164.4	34.0
Chen	162.7	69.0
Walker	79.4	71.8

Budget imbalances and infeasible outcomes: While the economic surplus the Chen and Kim mechanisms produced was, on average, only about 30% below the optimal level of E$ 243, the mechanisms experienced a much more serious failure: the budget was balanced in only five of the 360 plays in the Chen mechanism, and only twelve times out of 360 plays in the Kim mechanism. About half the time (54% and 51% respectively) there was a budget surplus: the participants were required to pay more in taxes than the cost of the public good. Since these excess taxes cannot be rebated to the participants without altering the mechanism, we must count them as an additional cost of the mechanism, thereby reducing the economic surplus the mechanism generates for its participants. Conversely, in the case of a budget deficit (the remaining 44% and 46% of plays, respectively), implementation of the mechanism's outcome requires an infusion of resources from external sources — again, an additional cost of implementing the mechanism.

Taking these additional costs into account, the economic surplus produced by the Kim mechanism was reduced, on average, by 38%, from E$ 164 to E$ 101. The magnitudes of the budget imbalances were far more serious in the Chen mechanism: in more than 90% of the 360 plays the budget imbalance was greater than E$ 100; it was more than E$ 1,000 in one-third of the plays; and it was occasionally more than E$ 10,000. Deducting the budget imbalances from the (direct) economic surplus $S(q)$ reduced the Chen mechanism's average (net) economic surplus to a *negative* E$ 1,051 (163 - 1214).

The budget is identically balanced in the Walker mechanism, but that mechanism's average economic surplus was still well below the Kim mechanism's average net economic surplus of E$ 101.

Individual rationality: We say that an outcome is *acceptable* to a participant if it leaves him at least as well off as he would be at the status quo outcome in which $q = 0$ and there are no taxes or transfers. When there is a natural status quo outcome like this, or where there is a need to guarantee participation in a mechanism, outcomes are often called *individually rational* if they are acceptable to every participant. Lindahl allocations are always individually rational, so each of the mechanisms in our experiment always produces an individually rational outcome at a Nash equilibrium.

But we've seen that these mechanisms never produced a Nash equilibrium. And in each of the mechanisms, out-of-equilibrium profiles (m_1, m_2, m_3) of messages may yield outcomes that are not individually rational. This occurred with considerable frequency in our experiment: 39% of the 1080 individual outcomes in the Chen mechanism were unacceptable to a participant, 11% were unacceptable in the Kim mechanism, and 29% were unacceptable in the Walker mechanism.

Summarizing: These three mechanisms all yield the Lindahl outcome at their equilibria, but in our experiment none of the mechanisms was ever in equilibrium. Disequilibrium is certainly not bad per se: if outcomes are not far from equilibrium, and if welfare is not far from what it would be in a good equilibrium, that would generally be considered a success. But the three Lindahl mechanisms we included in our experiment mostly failed, rather badly, to satisfy several criteria that we would generally regard as essential when designing an allocation mechanism. As we describe in the following section, we used these failures as a guide in an effort to design a mechanism that would be more robust to out-of-equilibrium behavior — a mechanism that would be relatively successful even if typically out of equilibrium.

The PQ Mechanism

In V&W (2017) we defined a mechanism, which we call the price-quantity or PQ mechanism, in which participants make quantity-and-price proposals (q_i, π_i). These proposals are the arguments of an outcome function φ that determines the level q at which a public good will be provided, as well as the amount t_i each participant i will pay to finance the public good. The price proposal π_i and the tax t_i may be any real numbers; if π_i is negative, $|\pi_i|$ is a proposed per-unit subsidy to be paid to i; if t_i is negative, $|t_i|$ is a proposed total payment to i.

We assume that each participant's maximum ability to pay (for example, his income or wealth) is observable and we denote it by $\mathring{y}_i$. The mechanism restricts participant i to proposals that satisfy $\pi_i q_i \leqq \mathring{y}_i$. For any $\mathring{y} \in \mathbb{R}_+$ we denote the set of all such proposals by $\psi(\mathring{y})$:

$$\psi(\mathring{y}) = \{(q,\pi) \in \mathbb{R}^2 \,|\, q \geqq 0 \text{ and } \pi q_{\leqq} \mathring{y}\},$$

and the set of all profiles of admissible proposals as Ψ:

$$\Psi = \times_{i=1}^{n} \psi(\mathring{y}_i).$$

We assume that the cost of providing q units of the public good[1] is $C(q) = cq$. The PQ mechanism's outcome function $\varphi : \Psi \to \mathbb{R}^{N+1}$ is defined as follows:

$$q = \begin{cases} \min\{q_1, ..., q_N\}, & \text{if } \sum_i \pi_i \geqq c \\ 0, & \text{otherwise;} \end{cases}$$

$$t_i = p_i q, \qquad \text{where} \qquad p_i = \frac{1}{N}c + \pi_i - \frac{1}{N}\sum_{j=i}^{n} \pi_j \qquad (i = 1, ..., N).$$

Thus, if the participants' price proposals π_i cover the cost of production $\left(i.e., \sum_i \pi_i \geqq c\right)$, then the mechanism produces the smallest quantity anyone has proposed. If the price proposals don't cover the cost, the mechanism produces zero. Consequently, if $q > 0$ then $p_i \leqq \pi_i$, and if $q = 0$ then $t_i = 0$. Therefore each participant never pays more than the amount $\pi_i q_i$ he has proposed.

The PQ Mechanism's Properties

We denote profiles $\big((q_1, \pi_1), ..., (q_N, \pi_N)\big)$ of proposals by ξ. For every profile ξ of proposals, whether it's an equilibrium or not, the mechanism's outcome $(q, \mathbf{t}) = (q, t_1, \ldots, t_N) = \varphi(\xi)$ has the following properties:

(P1) The budget is balanced — *i.e.*, $\sum_{i=1}^{N} t_i = C(q)$ — because $\sum_{i=1}^{N} p_i \equiv c$.

(P2) No participant pays more than his proposed price π_i per unit of the public good.

(P3) As a consequence of (P1) and (P2) and the fact that $\pi_i q_i \leqq \mathring{y}_i$, the outcome is both individually feasible and collectively feasible — *i.e.*, $t_i \leqq \mathring{y}_i$ for each $i = 1, \ldots, N$, and $C(q) \leqq \sum_1^N \mathring{y}_i$.

Assume that each participant's preference over outcomes is represented by a utility function $u_i(q, t_i)$ which is strictly quasiconcave, strictly increasing in q, and strictly decreasing in t_i. (Note that this is equivalent to saying the participant has a strictly

[1]For a more general cost function $C(q)$, c is replaced by $C(q)/\min\{q_1, ..., q_N\}$ in the equations defining the outcome function. Some of the properties described below do not hold for a nonlinear cost function.

quasiconcave, strictly increasing utility function over pairs $(q, y) \in \mathbb{R}^2_+$, where y_i is his after-tax dollar holdings, $\mathring{y}_i - t_i$.)

As noted above, we say that an outcome $(q, \mathbf{t}) = (q, t_1, \ldots, t_N)$ is **acceptable to** i if $u_i(q, t_i) \geqq u_i(0, 0)$ — *i.e.*, if participant i is at least as well off at the outcome $(q, \mathbf{t})$ as he is at the status quo outcome — and a proposal $\xi_i = (q, \pi_i)$ is acceptable to i if $u_i(q, \pi_i q) \geqq u_i(0, 0)$. For each i and each $\xi_i = (q, \pi_i) \in \psi(\mathring{y}_i)$, let $\varphi_i(\xi_i)$ denote the set of all outcomes that can occur if i chooses the proposal ξ_i:

$$\varphi_i(\xi_i) := \{(q, \mathbf{t}) \in \mathbb{R}^{N+1} \mid (q, \mathbf{t}) = \varphi(\widetilde{\xi}) \text{ for some } \widetilde{\xi} \in \Psi \text{ s.t. } \widetilde{\xi}_i = \xi_i \}.$$

We say that a proposal $\xi_i \in \psi(\mathring{y}_i)$ is **uniformly acceptable** to i if every outcome in $\varphi_i(\xi_i)$ is acceptable to i.

(P4) If preferences are quasiconcave, then under the outcome function φ every proposal $\xi_i = (q_i, \pi_i)$ that satisfies $u_i(q_i, \pi_i q_i) \geqq u_i(0, 0)$ is uniformly acceptable to player i. In other words, any proposal that's acceptable to a participant is uniformly acceptable to him. If he makes only proposals that are acceptable to him, then the outcome under φ (whether in equilibrium or not) will always be acceptable to him.

The properties (P1)-(P4) hold for all profiles of proposals and therefore for all outcomes of the mechanism, not merely for the equilibrium outcomes. This is in contrast to the three Lindahl mechanisms in the VLW experiment: although (P1) and (P3) hold for equilibrium outcomes in those mechanisms, (P1)-(P4) fail to hold in general. And indeed, in the VLW experiment the mechanisms' outcomes often violated these properties, generally by large amounts.

The PQ mechanism's equilibria also have several properties worth noting. Recall that an outcome is **individually rational** if it is acceptable to every participant $i = 1, \ldots, n$.

(P5) It follows from (P4) that a Nash equilibrium of the PQ mechanism is individually rational.

(P6) The Lindahl outcome is an equilibrium outcome.

The PQ mechanism has many Nash equilibria, in fact a continuum of them. (These are described in some detail in V&W (2017)). In particular, there are equilibria in which the public good level is zero. In order to gain some insight into the outcomes that participants in the mechanism will actually attain, we conducted an experiment.

An Experiment: The PQ Mechanism

In V&W (2018) we report on an experiment we conducted to compare the performance of the PQ mechanism with the results in the VLW experiment. In order to generate results that can be directly compared to the results in the VLW experiment, we used the same public-good problem: three participants, with the same valuation functions $v_i(x)$ for each $i = 1, 2, 3$ and the same cost function, $C(x) = 12x$. Therefore the unique Pareto level of the public good is the same, $\widehat{x} = 9$; the maximum possible economic surplus is $S(\widehat{x}) = E\$\,243$; and the Lindahl taxes are $t_1 = 36, t_2 = -18, t_3 = 90$. Each participant's surplus $v_i(\widehat{x}) - t_i$ at the Lindahl outcome is E$ 81. The experiment had 81 subjects, divided into 27 three-person groups.

We describe the results of this experiment along several dimensions, in each case comparing the results to those described above in the VLW experiment.

Equilibrium

The PQ mechanism's participants played equilibrium profiles in 282 of the 1080 plays (26%), and in 44% of the 270 later-period plays, from period 31 to period 40. Recall that the participants in the Lindahl mechanisms never played an equilibrium, out of 360 plays in each mechanism. The numbers are perhaps misleading, however, because the PQ mechanism has many equilbria while each of the other three mechanisms has only one equilibrium. The high frequency of equilibrium play in the PQ mechanism might be mostly due to nothing more than the presence of so many equilibria.

Nine of the twenty-seven groups attained one of the equilibria and continued to play that equilibrium in nearly every subsequent period. Each of these instances of equilibrium play produced public good levels of either 6 or 7 units, with E$216 or E$231 of economic surplus, somewhat less than the Pareto level of E$243. Clearly, none of these observed equilibria was the PQ mechanism's Lindahl equilibrium, since the public good levels they achieved were smaller than the Pareto public good level of 9 units.

Economic Surplus

Figure 2 depicts the average surplus attained, over all 40 periods, by each of the 27 groups in our experiment, as well as by each of the 9 groups in each of the three Lindahl mechanisms in the VLW experiment. Each of the graphs orders the 27 or 9 observed levels of surplus from smallest to largest, left to right — so the graphs are the empirical cdf's of the observed levels of surplus, with the cumulative frequencies (or percentiles) on the horizontal axis.

It's clear from the graphs in Figure 1 that the groups who did the worst in the Chen mechanism did much worse than the corresponding worst-performing groups in the PQ mechanism, and the groups who did the best in the Chen mechanism did slightly better than the corresponding best-performing groups in the PQ mechanism. The welfare distributions generated by the Kim and the PQ mechanisms were quite close, and the distribution of welfare produced by the Walker mechanism was significantly dominated by the PQ distribution. The mean surplus achieved by the PQ mechanism was E$ 175 and the standard deviation of the distribution is 28.8. The mean is not statistically different from the means for the Chen and Kim mechanisms that appear in Table 1. The standard deviation is lower than the standard deviation of 69 for the Chen mechanism at the 5% significance level, and is not statistically different than the Kim distribution's standard deviation of 34.

Measured by the direct economic surplus the mechanisms produced, the PQ mechanism seems to have performed at least as well as the Chen and Kim mechanisms, and clearly better than the Walker mechanism.

Budget Balance and Feasibility: Recall that the issues of budget imbalance and infeasibility of outcomes were a serious problem in the Chen and Kim mechanisms: the budget was almost never balanced in either mechanism, and when we took these costs into account in measuring the economic surplus the mechanisms produced, the reduction in surplus was significant for the Kim mechanism (reducing the surplus from E$ 164 to E$ 101, and overwhelming in the Chen mechanism (reducing the surplus from E$ 163 to negative E$ 1,051).

The property (P1) of the PQ mechanism — that the budget is always balanced, whether in or out of equilibrium — therefore appears to be an important advantage. The economic surplus the PQ mechanism produced was as large as, and no more variable than, the surplus produced by the Chen and Kim mechanisms directly. And when we take account of the additional costs imposed on mechanisms by budget imbalances, the net surplus of the other two mechanisms falls well below the E$ 175 surplus produced by the PQ mechanism.

Individual rationality: Recall that in the Chen and Kim mechanisms many of the outcomes were not individually rational. Because the PQ mechanism has the uniform acceptability property (P4) and each participant's valuation function is concave, a participant in the PQ mechanism, by always choosing a proposal that's acceptable to him, can ensure that the outcome will always make him at least as well off as the status quo. Only 32 of the 3240 proposals made by the 81 subjects in our experiment were not acceptable (less than one percent), and only one of the 1080 outcomes failed to be individually rational, by failing to be acceptable to only one participant (less than one-tenth of one percent).

Summarizing: Ignoring budget imbalances and infeasibility of outcomes in the Lindahl mechanisms, and in spite of the multiple non-optimal equilibria of the PQ mechanism, the PQ-mechanism performed at least as well as the Lindahl mechanisms we had examined in our earlier experiment. If we then take account of unbalanced budgets — and include their costs as reductions in welfare — the PQ mechanism clearly out-performed the Lindahl mechanisms.

Concluding Remarks

In the theory of mechanism design, equilibrium analysis has paid enormous dividends, illuminating myriad issues, from the possibility of providing economic agents with differing incentives, to the important roles of information and beliefs — all of which were anticipated by Hurwicz in the earliest stages of his development of the theory.

In *Putting Auction Theory to Work*, Milgrom almost exclusively puts mechanism design's *equilibrium theory* to work. But at the outset he points out that "the equilibrium analysis of game theory is an abstraction based on a sensible idea" which "relies on stark and exaggerated assumptions to reach theoretical conclusions that can sometimes be fragile." He lists assumptions about players perfectly maximizing, about players' information, and about their beliefs about other players' maximization, information, and beliefs, and points out that "these assumptions are extreme."

To Milgrom's list of assumptions we would add the "assumption" of equilibrium. Without denying the power and influence of the equilibrium assumption (like all of us, the authors have made careers from a reliance on it), we suggest that it would be fruitful to incorporate disequilibrium analysis into the theory as well. We mean not merely that we should ask whether disequilibria will converge over time, or how long convergence will take — *i.e.*, "Are we there yet?" Rather, we should recognize that we're never actually going to get there — it's the journey that matters, not the destination. As we've suggested above, we regard this idea as an extension of the "Wilson doctrine" that mechanisms should be "robust." The notion of **universal acceptability** that we introduced here, and which we applied to *all* behavior, disequilibrium as well as equilibrium, in the PQ mechanism for a public good, is a first attempt at this approach.

References

1. Chen, Y. (2002). "A family of supermodular Nash mechanisms implementing Lindahl allocations." *Economic Theory*, 19, 773-790.

2. Dubey, P. (1982). "Price Quantity Strategic Market Games. " *Econometrica*, 50, 111-126.

3. Hurwicz, L. (1979). "Outcome Functions Yielding Walrasian and Lindahl Allocations at Nash Equilibrium Points." *The Review of Economic Studies* 46, 217-224.

4. Hurwicz, L. and M. Walker (1990). "On the Generic Nonoptimality of Dominant-Strategy Allocation Mechanisms: A General Theorem that Includes Pure Exchange Economies." *Econometrica*, 58, 683-704.

5. Kim, T. (1993). "A Stable Nash mechanism implementing Lindahl allocations for quasi-linear environments." *Journal of Mathematical Economics*, 22, 359-371.

6. Milgrom, P. (2004). *Putting Auction Theory to Work*, Cambridge U. Press.

7. Van Essen, M., Lazzati, N., and Walker, M. (2012). "Out-of-Equilibrium Performance of Three Lindahl Mechanisms: Experimental Evidence," *Games and Economic Behavior*, 74, 366-381.

8. Van Essen, M. and Walker, M. (2017). "A Simple Market-Like Allocation Mechanism for Public Goods." *Games and Economic Behavior, 107,* 6-19.

9. Van Essen, M. and Walker, M. (2018). "An Experimental Evaluation of a Price-Quantity Mechanism for Public Goods." U. of Arizona working paper.

10. Walker, M. (1981). "A Simple Incentive Compatible Scheme for Attaining Lindahl Allocations." *Econometrica*, 49, 65-71.

11. Wilson, R. (1987). "Game Theoretic Anayses of Trading Processes. " *Advances in Economic Theory: Fifth World Congress*, ed. Truman F. Bewley, Cambridge U. Press, 33-70.

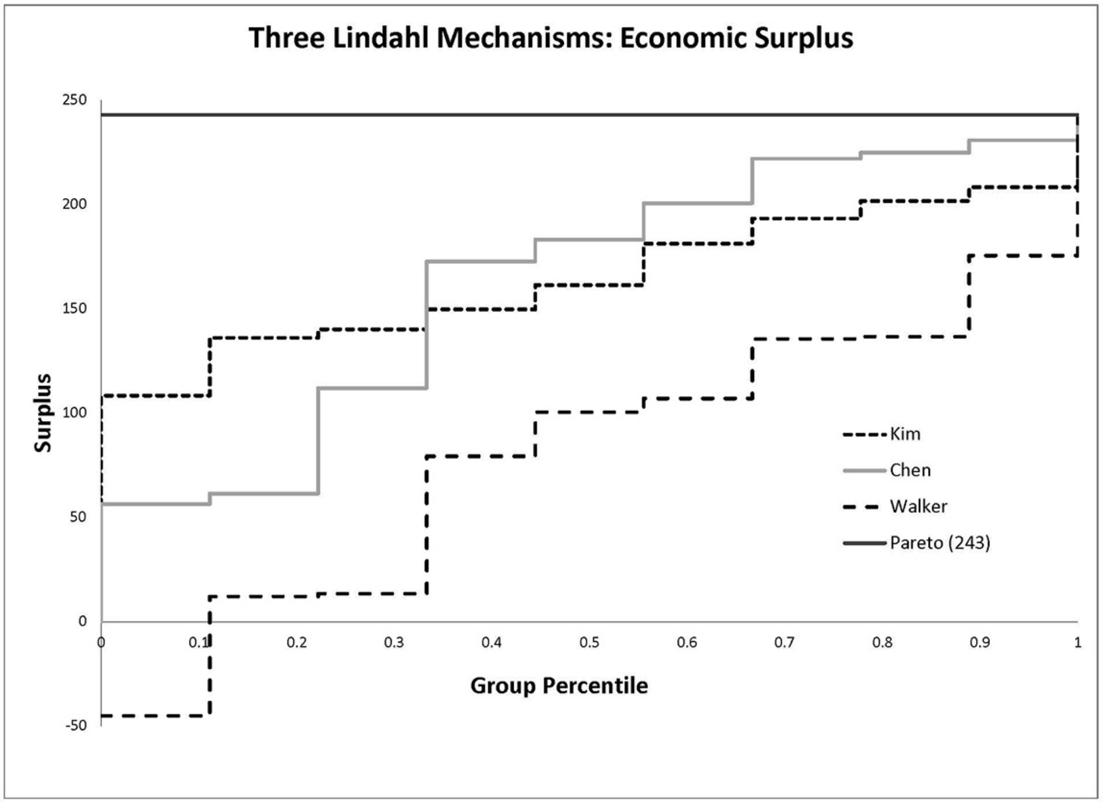
Three Lindahl Mechanisms: Economic Surplus
250
200
150
100
50
0
-50
Surplus
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Group Percentile
Kim
Chen
Walker
Pareto (243)

Figure 1

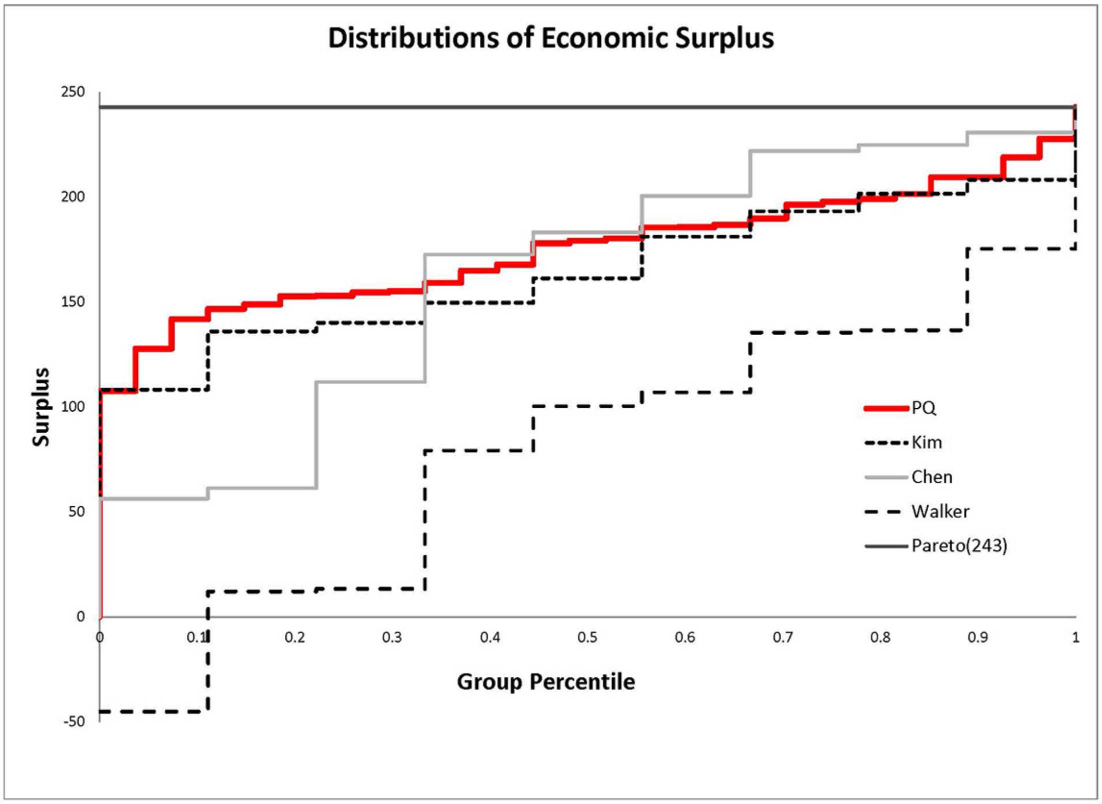
Distributions of Economic Surplus
250
200
150
100
50
0
-50
Surplus
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Group Percentile
PQ
Kim
Chen
Walker
Pareto(243)

Figure 2