General Equilibrium Rebound from Energy Efficiency Policies*

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I study the general equilibrium implications of large-scale policies to improve energy efficiency, which are being implemented around the world to mitigate climate change. Energy efficiency policies "rebound" when economic responses undercut their direct energy savings. I develop an analytic general equilibrium framework that allows improvements in energy efficiency to affect output prices and factor prices. I show that general equilibrium channels typically amplify rebound and make it more likely that improvements in efficiency end up increasing total energy use. General equilibrium channels are likely to be especially problematic when sectors targeted for efficiency improvements have a large value share of energy. In these cases, ignoring general equilibrium channels can severely bias benefit-cost analyses in favor of energy efficiency policies.

JEL: D58, H23, O33, Q43, Q58

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Many governments have adopted energy efficiency policies in order to reduce the greenhouse gas emissions that drive climate change and to reduce dependence on energy resources. The American Recovery and Reinvestment Act of 2009 provided nearly $20 billion for energy efficiency programs (Aldy, 2013). The U.S. Environmental Protection Agency's Clean Power Plan employs energy efficiency as one of the three "building blocks" through which the U.S. electric power sector will reduce its carbon emissions to 32% below 2005 levels by 2030. The European Union's 2012 Energy Efficiency Directive mandates a 20% improvement in energy efficiency by 2020. And many countries have programs that require improvements in the efficiency of their vehicles and appliances.

All of these policies are large-scale. Indeed, any efficiency policy that aims to matter for climate change must be large-scale. Such large-scale policies are likely to have general equilibrium consequences. While economists have long recognized the potential for general equilibrium responses to undercut efficiency policies' fuel savings (Jevons, 1865), nearly all formal analyses of these "rebound" effects have focused on partial equilibrium settings with exogenous prices for energy resources and other factors of production.[1] At one end, microeconomic analyses have emphasized how income and substitution effects can increase household energy consumption after an improvement in efficiency.[2] At the other end, neoclassical growth settings have emphasized how analogues of these income and substitution effects arise after improving the productivity of energy in the broader economy's production function (Saunders, 1992, 2000). Despite the theoretical literature's focus on partial equilibrium settings, computable general equilibrium models have suggested the potential for strong rebound effects through economy-wide "indirect" channels.[3]

I fill the gap in the theoretical literature by developing an analytically tractable general equilibrium framework for analyzing the implications of efficiency policies for resource use and emissions.[4] I provide intuitive expressions for general equilibrium rebound and disentangle the channels through which efficiency policies affect total resource use. The economy contains an arbitrary number of sectors that produce distinct consumption goods. Households have Dixit-Stiglitz preferences over these varieties of consumption goods. Each consumption good is produced competitively by combining a labor-capital aggregate with energy, using a constant elasticity of substitution technology. Each household supplies a single unit of the labor-capital aggregate to the production sector that offers the highest price.

[1] Reflecting on the potential importance of economy-wide rebound channels, Dimitropoulos (2007) notes the lack of a theoretical framework for understanding general equilibrium channels. Borenstein (2015) similarly calls for further research on channels for economy-wide rebound.

[2] There are several overviews of this microeconomic, partial equilibrium literature. See Greening et al. (2000), Sorrell and Dimitropoulos (2008), Sorrell et al. (2009), van den Bergh (2011), Gillingham et al. (2013), Borenstein (2015), and Gillingham et al. (2016).

[3] For instance, see the studies mentioned in Allan et al. (2009) and Turner (2013).

[4] The two other analytic general equilibrium settings are Wei (2007, 2010). Wei (2007) restricts attention to Cobb-Douglas functional forms for all production functions. Wei (2010) considers a setting with only a single consumption good.

Energy is produced by combining a sector-specific energy conversion technology with raw energy resources, which are supplied competitively. An efficiency policy improves the quality of some sector's energy conversion technology, which affects prices and activity throughout the economy.[5]

An engineering estimate of the effects of an efficiency policy would hold energy service production fixed and calculate the energy resources displaced by the improvement in efficiency. "Rebound" is the percentage of these engineering savings lost through economic responses. A partial equilibrium analysis of rebound holds the prices of consumption goods, energy resources, and the labor-capital aggregate fixed. In this case with fixed prices, I show the result familiar from previous literature (e.g., Saunders, 1992; Sorrell and Dimitropoulos, 2008): rebound is proportional to the elasticity of substitution between energy and non-energy inputs. This elasticity captures how firms substitute towards the energy input when improved technology reduces its effective cost. When energy and non-energy inputs are gross substitutes in production (i.e., when this elasticity is greater than 1), rebound is greater than 100%. In this case, an efficiency policy is said to "backfire," actually increasing consumption of energy resources.

In general equilibrium, all prices adjust to the improved energy conversion technology. Improving the technology in one sector reduces the cost of producing that sector's consumption good. Households substitute towards that consumption good as its price falls. This substitution increases demand for both energy and non-energy inputs to production in that same sector with improved technology. This increased demand works directly to increase total resource use. However, the increased factor demand also raises the price of the labor-capital aggregate, which has two effects. First, the cost of producing the other consumption goods increases, which reduces demand for those goods and thus for energy resources. Second, the higher price for the labor-capital aggregate increases households' income and thereby increases demand for all consumption goods. The net effect on resource use depends on the resource intensity of the sector with improved technology and on the elasticities of substitution throughout the economy. I show that these general equilibrium channels typically increase rebound. Further, I show that they always increase rebound in the special case where all good-producing sectors initially have the same production technology.

I connect general equilibrium rebound to parameters that can be estimated in future empirical work and used in policy evaluations. In particular, I show that the general equilibrium component of rebound depends on three terms: it grows with the value share of resources in the sector with improved technology and with the elasticity of substitution between the various consumption goods, and it becomes small when the elasticity of substitution between energy and non-energy inputs is large in the sector with improved technology. The value share determines the degree to which improved technology reduces the price of that sector's consumption good, and the two elasticities determine how factor demand scales with

[5]One can also interpret the present setting's "energy" as energy services (such as heating or lighting) and the present setting's "energy conversion technology" as the efficiency of energy service production.

the price of the consumption good. General equilibrium channels can be safely ignored in sectors with a small value share of resources.[6]

Most troublingly, I show that general equilibrium channels can make an efficiency policy backfire (i.e., increase total resource use) even for arbitrarily small elasticities of substitution between energy and non-energy inputs. In contrast to received wisdom, the elasticity of substitution is not a reliable guide to the likelihood of backfire. This result is important because much empirical work has suggested that the elasticity of substitution between energy and non-energy inputs is less than unity, which would make backfire irrelevant in a partial equilibrium analysis. Instead, my results suggest that the potential for backfire is an empirically relevant question after all, which is consistent with the many computable general equilibrium models that have in fact found backfire when analyzing particular policies (Semboja, 1994; Grepperud and Rasmussen, 2004; Glomsrød and Taoyuan, 2005; Hanley et al., 2006; Allan et al., 2009; Hanley et al., 2009). Numerical examples illustrate how general equilibrium effects greatly expand the set of elasticities consistent with backfire and can even make backfire an especially severe problem for elasticities much smaller than unity.

Finally, I explore how to target an efficiency policy. In actual economies, different sectors have different initial technologies and different elasticities of substitution. I show that a policymaker who aims to reduce resource use through improved efficiency should target the sector with the smallest elasticity of substitution between energy and non-energy inputs. Here, partial and general equilibrium analyses give the same recommendation, though they can predict very different changes in resource use and thus lead to very different conclusions when used in cost-benefit assessments. I also show that a policymaker should generally target the least efficient sector when energy and non-energy inputs are substitutes and should target the most efficient sector otherwise. Intuitively, the policymaker can minimize the general equilibrium component of rebound by targeting sectors in which the value share of resources is small. The value share of resources increases in the quality of energy conversion technology if and only if energy and non-energy inputs are substitutes. Taken together, these results suggest that efficiency policies are most likely to achieve environmental goals when applied to sectors that are already relatively efficient and that also have a small elasticity of substitution between energy and non-energy inputs.

This paper extends a literature that demonstrates the perils of ignoring general equilibrium consequences in policy evaluations. Heckman et al. (1998, 1999) argue that the conventional econometric approach to treatment effects assumes away interactions among affected agents through market channels. They show that implementing conventional estimates of treatment effects can severely bias evaluations of education policies. Acemoglu (2010) similarly cautions against applying reduced-form econometric estimates to evalua-

[6]Historical evidence supports the importance of the value share of resources for the possibility of backfire. Rosenberg (1994)[Chapter 9, p. 165] observes, "Historically, new technologies that improved energy efficiency have often led to a significant increase, and not to a reduction, in fuel consumption. This has been especially true in energy-intensive sectors where fuel costs have constituted a large proportion of total costs."

tions of development policies. Smith and Carbone (2007) and Carbone and Smith (2008) show that benefit-cost analyses of tax policies can be severely distorted by ignoring general equilibrium interactions with public goods. Reinforcing and extending these examples from labor, development, and public economics, I show that general equilibrium channels can also be critical when evaluating the large-scale policies now being adopted to address climate change. Recent econometric work has estimated the energy savings generated by treating households with more efficient appliances or with improvements in thermal efficiency (e.g., Dubin et al., 1986; Davis, 2008; Jacobsen and Kotchen, 2011; Davis et al., 2014; Levinson, 2016). My results suggest that economists should exercise caution before using these estimates in evaluations of larger-scale policies to improve energy efficiency.

This paper also extends several recent literatures exploring the unintended consequences of environmental policies. First, the "green paradox" literature considers how energy policies can backfire by changing resource extractors' incentives to conserve resources for the future (e.g., Sinn, 2008; Gerlagh, 2011). I abstract from dynamic considerations in order to demonstrate static, general equilibrium channels for backfire. Second, several papers have explored how environmental regulations that constrain the energy intensity or emission intensity of production can backfire (e.g., Helfand, 1991; Holland et al., 2009; Fullerton and Heutel, 2010; Lemoine, 2016). These effects arise because an intensity constraint implicitly combines an output subsidy with a tax on energy or emissions. I instead explore the consequences of more common policies that directly incentivize the adoption or development of technologies that reduce the energy intensity of production, without constraining firms' profit maximization problems. Third, other literature on the general equilibrium consequences of environmental policies has explored the potential for leakage between sectors or regions (e.g., Copeland and Taylor, 2004; Baylis et al., 2014). I develop a more textured model of energy use and production that allows me to answer questions about increasingly common policies to improve energy efficiency.

The next section describes the setting. Section 2 derives the equilibrium prices and allocation. Section 3 contains the partial equilibrium analysis. Section 4 analyzes general equilibrium rebound graphically and then theoretically, first for a case with symmetric sectors and then for the general case with heterogeneous sectors. Section 5 contains numerical examples. The final section concludes. The first appendix connects the analysis to a case in which energy services are a direct input to utility. The second appendix contains proofs and additional analysis.

1 Setting

Proceeding formally, sector i produces quantity of c_i of its consumption good, with $i \in \{1, ..., N\}$. Households obtain utility from consuming these goods:

$$u(C), \text{ where } C \triangleq \left(\sum_{i=1}^{N} c_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Utility $u(\cdot)$ is monotonically increasing in the consumption index C. $\epsilon > 1$ denotes the elasticity of substitution between the different varieties of consumption good. The price of each good is p_i.

Each consumption good is produced competitively using quantity X_i of labor-capital aggregate and quantity E_i of energy:

$$c_i = \left(\kappa X_i^{\frac{\sigma_i-1}{\sigma_i}} + (1-\kappa) E_i^{\frac{\sigma_i-1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i-1}}.$$

The production function has a constant elasticity of substitution $\sigma_i \in (0, \epsilon)$, with $\sigma_i \neq 1$. $\kappa \in (0, 1)$ is the distribution parameter. Energy E_i is produced by combining an energy conversion technology of quality A_i with a quantity R_i of energy resources:

$$E_i = A_i R_i.$$

We refer to energy and energy conversion technologies for simplicity, but one can also interpret E_i as energy services (such as heating or lighting) and A_i as the efficiency of energy service production. Throughout, α_{Xi} and α_{Ri} will denote the value share of X_i and R_i, respectively, in sector i.

The same resources are used in each sector. The total quantity of resources consumed is $R = \sum_{i=1}^{N} R_i$. In equilibrium, each sector pays price p_R for each unit of the resource. Resources are supplied isoelastically and competitively:[7]

$$R = \Psi p_R^{\psi},$$

with $\Psi, \psi > 0$. We will consider the implications of marginal improvements in some A_i for total resource consumption R.[8]

[7]Households own the non-energy factor of production but not the energy resources. We can therefore isolate income effects arsing through interactions between efficiency policies and non-energy inputs to production. This setting is consistent with an economy that imports a large share of its energy resources, as in many of the developed countries that are using efficiency policies to combat climate change.

[8]Much literature has explored the reasons why households and firms appear, by some calculations, to underinvest in efficiency. The present setting does not require optimal investment in efficiency technologies. Instead, it only requires that firms maximize profits conditional on using some particular technologies.

There is a continuum of households, of measure L. Each household is endowed with one unit of the labor-capital aggregate and sells it to some sector i. In equilibrium, each sector pays price p_X for each unit of labor-capital aggregate. Each household's budget constraint is $\sum_{i=1}^N p_i c_i \leq p_X$. Households will choose to sell all of their endowment. Therefore $L = \sum_{i=1}^N X_i$.

We study market equilibria.

Definition 1. *An* equilibrium *is given by consumption good prices ($\{p_i\}_{i=1}^N$), a price for the labor-capital aggregate (p_X), a price for resources (p_R), demands for inputs ($\{X_i, R_i\}_{i=1}^N$), and demands for consumption goods ($\{c_i\}_{i=1}^N$) such that: (i) (X_i, R_i) maximizes profits of producers of consumption good i, (ii) $\{c_i\}_{i=1}^N$ maximizes household utility, (iii) firms make zero profits, and (iv) the prices p_X, p_R, and $\{p_i\}_{i=1}^N$ clear the markets for the non-energy input, the resource input, and each consumption good, respectively.*

The equilibrium prices clear all factor markets, all firms maximize profits within competitive markets, and households maximize utility subject to their budget constraint.[9]

2 Equilibrium Prices and Allocations

Each household solves the following maximization problem:

$$\max_{\{c_i\}_{i=1}^N} \left\{ u\left(\left(\sum_{i=1}^N c_i^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \right) \right\}, \text{ subject to } \sum_{i=1}^N p_i c_i \leq p_X.$$

Letting λ be the shadow value of the budget constraint, the first-order condition for c_i is

$$\frac{\lambda p_i}{u'(C)} = \left(\frac{c_i}{C}\right)^{-\frac{1}{\epsilon}}.$$

Therefore

$$\frac{p_i}{p_j} = \left(\frac{c_i}{c_j}\right)^{-\frac{1}{\epsilon}}.$$

Let P be the ideal price index, so that $\sum_{i=1}^N p_i c_i = P\,C$. Households' first-order condition for C implies that $P = u'(C)/\lambda$. We choose the price index as the numeraire: $P = 1$. The household budget constraint then implies that $C = p_X$ in equilibrium. Demand for good i becomes

$$c_i = \left(\frac{p_i}{P}\right)^{-\epsilon} C = p_i^{-\epsilon} p_X.$$

[9] Note that the zero-profit condition is not a restriction, since it is actually implied by profit maximization, market-clearing, and the constant returns to scale production functions.

Now consider the input mix chosen by firms in sector i. Firms solve:

$$\max_{X_i,R_i}\left\{p_i\left(\kappa X_i^{\frac{\sigma_i-1}{\sigma_i}}+(1-\kappa)E_i^{\frac{\sigma_i-1}{\sigma_i}}\right)^{\frac{\sigma_i}{\sigma_i-1}}-p_X X_i-p_R R_i\right\}.$$

The first-order conditions are:

$$p_X = p_i\kappa\left(\frac{X_i}{c_i}\right)^{-\frac{1}{\sigma_i}}, \tag{1}$$

$$p_R = p_i(1-\kappa)A_i^{\frac{\sigma_i-1}{\sigma_i}}\left(\frac{R_i}{c_i}\right)^{-\frac{1}{\sigma_i}}. \tag{2}$$

Rearranging the first-order conditions to solve for X_i and R_i, we obtain the zero-profit condition required by competitive markets:

$$p_i=\left(p_X^{1-\sigma_i}\kappa^{\sigma_i}+p_R^{1-\sigma_i}A_i^{\sigma_i-1}(1-\kappa)^{\sigma_i}\right)^{\frac{1}{1-\sigma_i}}. \tag{3}$$

This condition pins down each output price $p_i(p_X,p_R)$ as a function of the input prices p_X and p_R.

Rearranging the firms' first-order conditions to obtain demand for X_i and R_i, substituting for c_i from households' first-order conditions, and using $P=1$ and $C=p_X$, we have:

$$X_i=\left(\frac{\kappa}{p_X}\right)^{\sigma_i}p_i^{\sigma_i-\epsilon}p_X, \tag{4}$$

$$R_i=A_i^{\sigma_i-1}\left(\frac{1-\kappa}{p_R}\right)^{\sigma_i}p_i^{\sigma_i-\epsilon}p_X. \tag{5}$$

Market-clearing for the labor-capital aggregate implies that

$$L=\sum_{i=1}^{N}\left(\frac{\kappa}{p_X}\right)^{\sigma_i}p_i^{\sigma_i-\epsilon}p_X.$$

We denote excess demand for the labor-capital aggregate as

$$D^X(p_R,p_X)=\sum_{i=1}^{N}\left(\frac{\kappa}{p_X}\right)^{\sigma_i}p_i(p_X,p_R)^{\sigma_i-\epsilon}p_X-L.$$

Market-clearing for resources implies that

$$\Psi p_R^{\psi}=\sum_{i=1}^{N}A_i^{\sigma_i-1}\left(\frac{1-\kappa}{p_R}\right)^{\sigma_i}p_i^{\sigma_i-\epsilon}p_X.$$

We denote excess demand for resources as

$$D^R(p_R, p_X) = \sum_{i=1}^{N} A_i^{\sigma_i - 1} \left(\frac{1-\kappa}{p_R}\right)^{\sigma_i} p_i(p_X, p_R)^{\sigma_i - \epsilon} p_X - \Psi p_R^{\psi}.$$

The equilibrium p_R and p_X set each excess demand to zero.

We are interested in equilibria that are stable in a tâtonnement sense. Let the tâtonnement adjustment process be such that we find an equilibrium by increasing p_R if and only if $D^R > 0$ and by increasing p_X if and only if $D^X > 0$. The steady state reached by this process is an equilibrium of our economy. By Lyapunov's Theorem of the First Approximation, the tâtonnement adjustment process is locally asymptotically stable around the equilibrium if $\frac{\partial D^R}{\partial p_R} + \frac{\partial D^X}{\partial p_X} < 0$ and $\frac{\partial D^R}{\partial p_R}\frac{\partial D^X}{\partial p_X} - \frac{\partial D^R}{\partial p_X}\frac{\partial D^X}{\partial p_R} > 0$ around the equilibrium.[10]

Throughout, we assume that, around an equilibrium, $\sigma_i > \sigma_i^L \triangleq \epsilon - \frac{\epsilon - 1}{\alpha_{Ri}}$ for all i, which ensures that $\partial D^X / \partial p_X < 0$. Note that $\sigma_i^L \leq 1$, with $\sigma_i^L < 0$ if α_{Ri} is sufficiently small. The following proposition establishes sufficient conditions under which the equilibrium is stable:

Proposition 1. *The equilibrium is stable if, for all $i \in \{1, ..., N\}$, $\sigma_i > \sigma_i^L$ and any of the following hold:*

i $\sigma_i \geq \epsilon - 1$,

ii $\sigma_i = \sigma_j$ *for all* $j \in \{1, ..., N\}$ *and* $\sigma_i \leq \epsilon - \frac{\epsilon - 1}{\alpha_{Xi}} < 1$, *or*

iii $\sigma_i = \sigma_j$ *and* $A_i = A_j$ *for all* $j \in \{1, ..., N\}$.

Proof. See appendix. □

The equilibrium is generally stable when each sector has the same technology A_i and the same elasticity of substitution σ_i. When sectors have heterogeneous technologies and elasticities of substitution, equilibria are stable if $\sigma_i > \max\{\sigma_i^L, \epsilon - 1\}$, which we will see has an intuitive graphical interpretation. We henceforth restrict attention to stable equilibria.

3 Partial Equilibrium Rebound

We now consider the implications of a 1% improvement in the efficiency of energy production in some sector k.

[10]Formally, let the adjustment process be $\dot{p}_R = H_R(D^R)$ and $\dot{p}_X = H_X(D^X)$, where dot notation indicates a time derivative, $H_i(0) = 0$, and $H_i'(\cdot) > 0$, for $i \in \{R, X\}$. The stated conditions on the partial derivatives of D^R and D^X ensure that the eigenvalues of the linearization of this system are negative.

First, the simplest "engineering" calculation of the implications of an improvement in energy conversion technology holds total production of c_k and E_k fixed. Since $R_k = E_k/A_k$, we have

$$Savings^{eng} \triangleq -A_k \left.\frac{\mathrm{d}R_k}{\mathrm{d}A_k}\right|_{E_k \text{ fixed}} = R_k.$$

The engineering calculation predicts that a 1% improvement in the efficiency of energy conversion leads to a 1% reduction in resource use.

Economists have noted that improving the efficiency of energy conversion lowers the relative price of energy inputs, which leads profit- or utility-maximizing agents to increase their use of energy inputs. This substitution towards energy inputs is called rebound and is typically analyzed in a partial equilibrium setting in which the prices of resource inputs to energy production, of non-energy inputs to consumption-good production, and of consumption good outputs are held fixed.[11] Formally, equation (5) implies:

$$A_k \frac{\mathrm{d}R_k}{\mathrm{d}A_k} =(\sigma_k - 1)R_k + (\sigma_k - \epsilon)\frac{A_k}{p_k}\frac{\mathrm{d}p_k}{\mathrm{d}A_k}R_k - \sigma_k \frac{A_k}{p_R}\frac{\mathrm{d}p_R}{\mathrm{d}A_k}R_k + \frac{A_k}{p_X}\frac{\mathrm{d}p_X}{\mathrm{d}A_k}R_k. \tag{6}$$

The partial equilibrium calculation does not allow p_R, p_X, or p_k to change with A_k, so that

$$Savings^{PE} \triangleq - A_k \left.\frac{\mathrm{d}R_k}{\mathrm{d}A_k}\right|_{p_R, p_X, p_k \text{ fixed}} = (1 - \sigma_k)R_k.$$

Partial equilibrium rebound, as a fraction of the no-rebound or "engineering" savings from an improvement in energy efficiency, is then

$$Rebound^{PE} \triangleq \frac{Savings^{eng} - Savings^{PE}}{Savings^{eng}} = \sigma_k.$$

Partial equilibrium rebound is equal to σ_k, a result familiar from many studies (e.g., Saunders, 1992; Sorrell and Dimitropoulos, 2008). This analysis suggests that a 1% improvement in energy efficiency most strongly reduces resource use when it targets a sector with high resource use (large R_k) and a low elasticity of substitution between energy and non-energy inputs (small σ_k). We see that partial equilibrium rebound can never be negative, which means that the engineering calculation is an upper bound on partial equilibrium energy savings. Partial equilibrium rebound goes to zero only as $\sigma_k \to 0$, in which case the firm has a Leontief production function and so has no scope to adjust its input mix. When $\sigma_k > 1$, partial equilibrium rebound is greater than 100%, so that improving energy efficiency actually increases resource use ("backfires").

[11] In some settings, energy services are modeled as a direct input to utility. In these cases, resource prices are indeed held fixed when analyzing rebound, but households can substitute towards energy services. See the appendix for an analysis of such a setting.

Figure 1 graphically describes the partial equilibrium effect. It plots the combinations of R_k and X_k that generate a given quantity of output c_k, and it also plots the budget line. Prior to the improvement in energy efficiency, the firm's profit-maximizing point is at A, where the isoquant is tangent to the budget line. Improving the efficiency of energy conversion technology changes the isoquant to the dashed line. The improvement in efficiency shifts the frontier by more in regions of heavy resource use. The engineering calculation of the change in resource use holds X_k fixed, so it finds the point B that is directly below point A but on the new isoquant and on a lower budget line. The vertical distance between points A and B defines the resource savings. However, the partial equilibrium calculation recognizes that the firm will reoptimize its input mix to return to a point of tangency. Because this calculation holds input prices constant, the budget line's slope does not change.[12] As $\sigma_k \to 0$ (left), point B is also the point of tangency with the new isoquant. For larger σ_k (right), the new point of tangency (labeled C) is to the left and above point B. The vertical distance between points B and C determines partial equilibrium rebound. As σ_k becomes larger, the isoquant becomes flatter and the vertical distance between points B and C grows. For $\sigma_k > 1$ (not pictured), point C is above point A, in which case rebound is greater than 100% (a case of "backfire").

4 General Equilibrium Rebound

The previous, partial equilibrium analysis held factor and output prices fixed and asked how resource use changed in the sector that experienced the improvement in energy efficiency. However, pollution is often related to the total change in resource use, including changes in other sectors induced by changes in factor and output prices. We now consider this total, general equilibrium change in resource use.

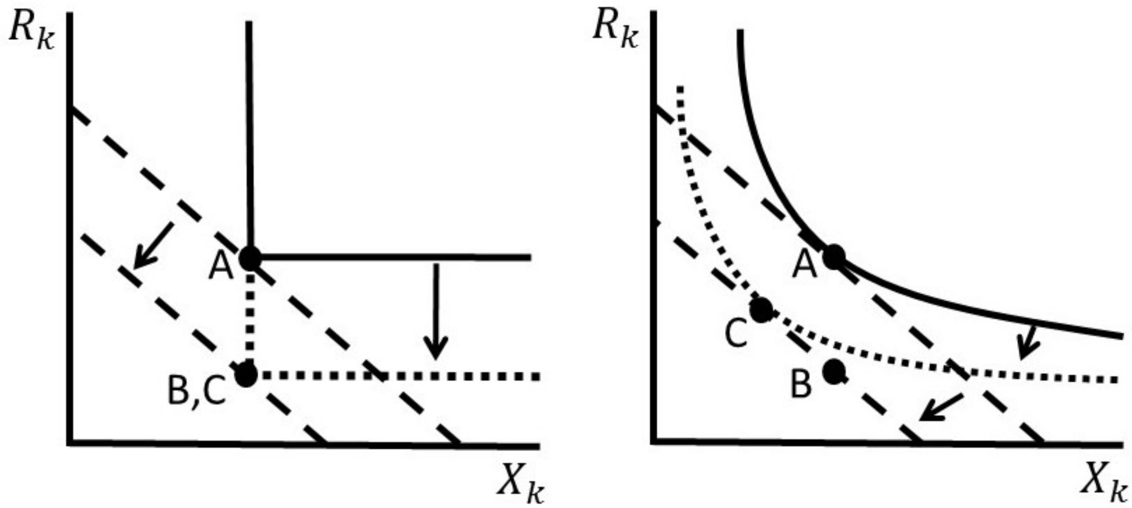
From equation (6), we have the total change in resource use from a 1% improvement in sector k's efficiency of energy conversion as:

$$A_k \frac{\mathrm{d}R}{\mathrm{d}A_k} = \sum_{i=1}^{N} \frac{\mathrm{d}R_i}{\mathrm{d}A_k} A_k = (\sigma_k - 1) R_k - \sum_{i=1}^{N} (\epsilon - \sigma_i) \frac{A_k}{p_i} \frac{\mathrm{d}p_i}{\mathrm{d}A_k} R_i - \frac{A_k}{p_R} \frac{\mathrm{d}p_R}{\mathrm{d}A_k} \sum_{i=1}^{N} \sigma_i R_i + \frac{A_k}{p_X} \frac{\mathrm{d}p_X}{\mathrm{d}A_k} R.$$

Define $\theta_{Y,Z}$ be the elasticity of Y with respect to Z: $\theta_{Y,Z} \triangleq \frac{\mathrm{d}Y}{\mathrm{d}Z} \frac{Z}{Y}$. Using this definition and noting that $\theta_{p_R,A_k} = \frac{1}{\psi} \theta_{R,A_k}$, we have:

$$\theta_{R,A_k} = \frac{(\sigma_k - 1)\frac{R_k}{R} + \sum_{i=1}^{N} \overbrace{\left[\theta_{p_X,A_k} - \epsilon\, \theta_{p_i,A_k}\right]}^{\theta_{c_i,A_k}} \frac{R_i}{R} + \sum_{i=1}^{N} \sigma_i\, \theta_{p_i,A_k} \frac{R_i}{R}}{1 + \frac{1}{\psi} \sum_{i=1}^{N} \sigma_i \frac{R_i}{R}}.$$

[12] Being on a lower budget line indicates that the zero-profit condition is no longer satisfied. The general equilibrium analysis will allow output and input prices to change, after which the zero-profit condition will again hold.



(a) $\sigma_k = 0$ (b) $\sigma_k \in (0, 1)$

Figure 1: Improving the quality of energy conversion technology A_k changes the isoquants of sector k's production technology from the solid line to the dotted line. Point A indicates the initial equilibrium. The gap between point A and point B is the no-rebound calculation of resource savings, and the gap between point B and point C defines partial equilibrium rebound.

We see that the elasticity of R with respect to A_k depends on four terms in the numerator.[13] The first term is the partial equilibrium effect that we analyzed before. When $\sigma_k > 1$, this first term is positive, so that improving the quality of energy conversion technology increases resource use by inducing sufficient substitution towards energy within sector k. The second term (with brackets) describes how improved efficiency in sector k changes demand for each sector's consumption good and thereby changes each sector's demand for resources. This change has two pieces. The first term in brackets captures how greater production of product i changes real household earnings p_X. When these income effects are positive ($\theta_{p_X,A_k} > 0$), this change increases demand for every consumption good c_i and thus increases demand for resource inputs in each sector i. This term will depend on how changing A_k changes demand for the non-energy input X, which will in turn depend on how changing A_k changes output prices. The second term in brackets captures how households substitute towards the newly abundant consumption good c_k, which reduces demand for other goods c_i and raises their equilibrium prices. This household substitution effect works to reduce demand for resource inputs in sectors $i \neq k$ and to increase demand for resource inputs in sector k. It is especially strong when ϵ is large.[14] Finally, the last term in the numerator captures how resource use changes with the price of output in each sector. This term is always dominated by the second term in brackets because $\epsilon > \sigma_i$.

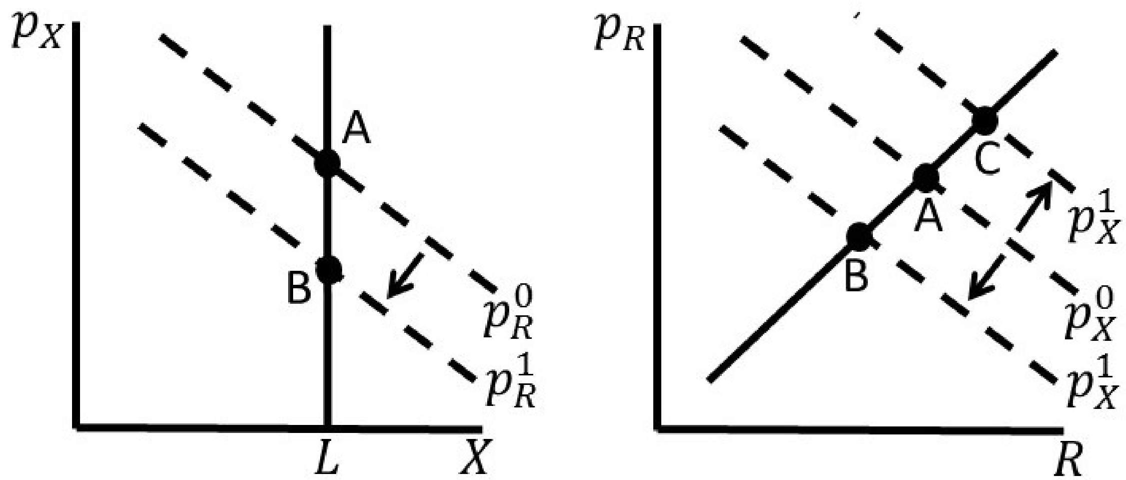
We see that the total change in resource use depends on how multiple prices change and on how these prices affect resource demand in each sector. We next develop graphical intuition for this general equilibrium setting. We then formally analyze rebound when all sectors have the same elasticity of substitution and the same initial technology. Finally, we formally analyze rebound when sectors differ in elasticities of substitution and/or initial technologies.

4.1 Graphical Analysis

The graphical analysis proceeds in three steps. Begin with the left panel of Figure 2. It plots supply (solid) and demand (dashed) of the non-energy input X. Supply is perfectly inelastic, fixed at L. Demand is depicted conditional on a resource price p_R. From equation (4), raising p_X has three effects on demand for each X_i. First, it reduces demand directly as the firm substitutes towards the energy input. Second, the increased input cost translates into

[13] Partial equilibrium analyses of rebound effects commonly assume that resource supply is perfectly elastic. The denominator here derives from θ_{p_R,A_k}. It shrinks the magnitude of any change in resource use when resource supply is not perfectly elastic ($\psi < \infty$). As $\psi \to 0$, resource supply becomes perfectly inelastic, and the denominator then ensures that the total change in resource use is zero.

[14] Previous partial equilibrium analyses of rebound effects sometimes include this channel (e.g., Chan and Gillingham, 2015): when households directly consume energy services, households can substitute towards energy services when they are gross substitutes with other inputs to utility. In most of these settings, the household maximizes over energy services and a non-energy input to utility, in which case $R_i = 0$ for $i \neq k$ and this channel clearly works to amplify rebound. See the appendix for a formal analysis.



(a) X factor market

(b) R factor market

Figure 2: Equilibrium in each factor market equates supply (solid) and demand (dashed). Point A depicts equilibrium at the initial price of the other factor (superscript 0). In the X factor market (left), increasing p_R shifts demand inward and changes the equilibrium to point B. In the R factor market (right), increasing p_X shifts demand inward when input cost effects dominate (changing the equilibrium to point B) and shifts demand outward when income effects dominate (changing the equilibrium to point C).

a higher output price at any level of production, which reduces consumption c_i and thus reduces demand for X_i. Third, increasing p_X increases household income, which increases demand for each consumption good. The first two effects argue for a downward-sloping demand curve, and the third argues for an upward-sloping demand curve. Our assumption that $\sigma_i > \sigma_i^L$ ensures that demand for X_i is downward-sloping, even after accounting for income effects.

Now consider how a change in the resource price p_R affects this demand curve. From equation (3), increasing the resource price p_R increases the output price p_i required for any given p_X, which reduces equilibrium consumption of c_i and thus reduces demand for X_i. Increasing p_R therefore shifts the demand curve inward, changing the equilibrium from a point such as A to a point such as B. p_X must fall as we increase p_R.

The right panel of Figure 2 depicts the second step in the graphical analysis. It is the same type of graph as in the left panel, but now depicting the resource market for a given p_X. Here the supply curve is upward-sloping rather than perfectly inelastic. And because households do not derive income from resource extraction, the demand curve is unambiguously downward-sloping. Now consider how an increase in p_X changes demand for R. As in the previous analysis, an input cost effect works to shift demand for each R_i

inward. Equation (5) shows that this effect is large when $\epsilon - \sigma_i$ is large: large ϵ means that consumers respond to a higher price of any good i by substituting especially strongly towards the other goods, and small σ_i means that demand for resources in sector i tracks production closely. However, equation (5) shows that an increase in p_X also shifts demand for each R_i outward, through an income effect. The income effect dominates in market i if and only if $\sigma_i > \epsilon - \frac{1}{\alpha_{Xi}} \triangleq \tilde{\sigma}_i$.[15] When the input cost effect dominates, raising p_X shifts the equilibrium from a point like A to a point like B, so that p_R declines as p_X increases. However, when the income effect dominates, raising p_X shifts the equilibrium from a point like A to a point like C, so that p_R increases as p_X increases.

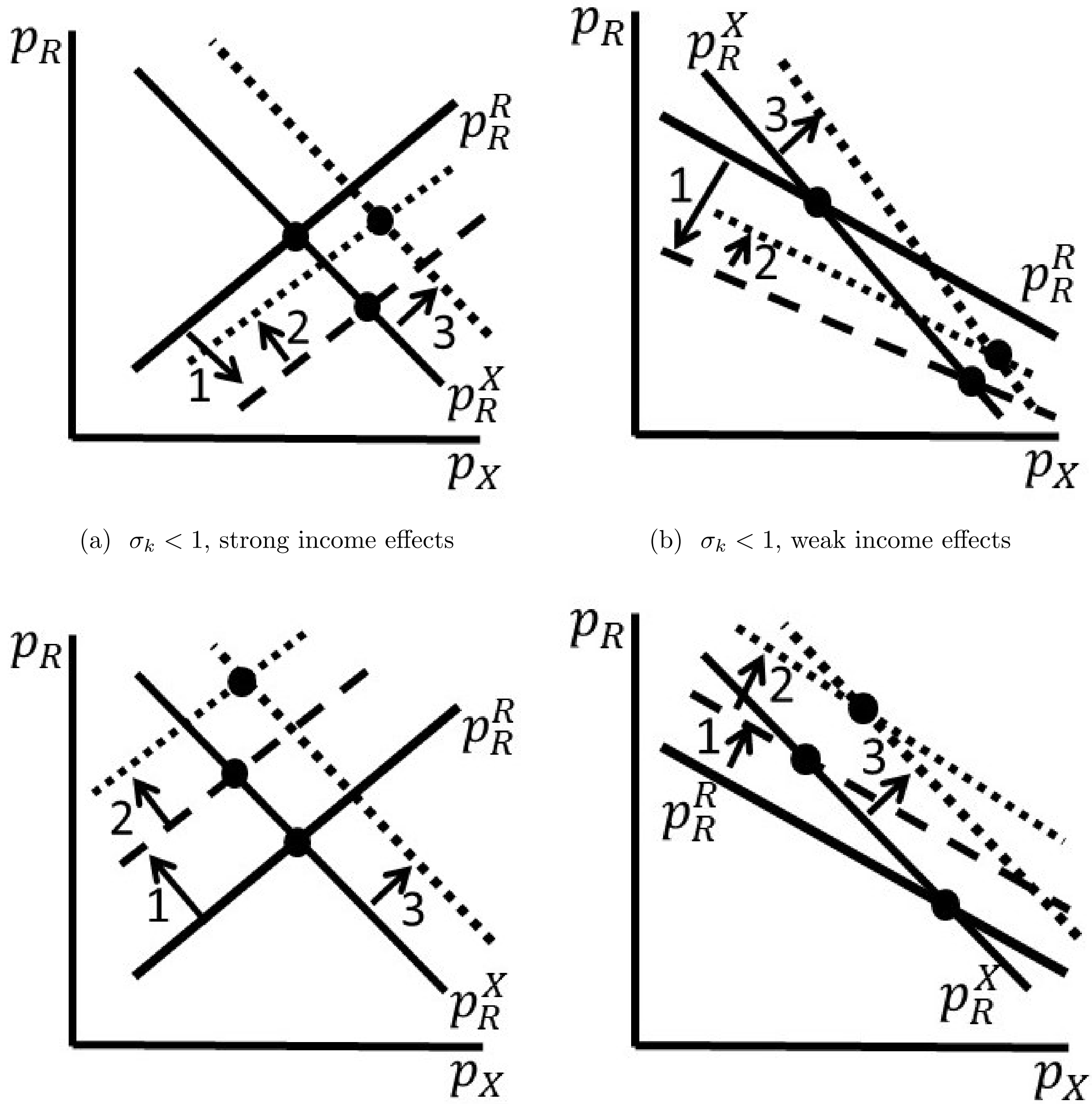
We can now put these pieces together in order to consider equilibrium prices and analyze how the equilibrium allocation responds to a change in the efficiency of energy conversion in some sector k. We have described two relationships between p_X and p_R. A first one, from the left panel of Figure 2, defines p_R as a downward-sloping function of p_X. We label this $p_R^X(p_X)$ because it sets excess demand for X to zero. The second one, from the right panel of Figure 2, defines p_R as a potentially upward- or downward-sloping function of p_X. We label this $p_R^R(p_X)$ because it sets excess demand for R to zero. The left panels of Figure 3 depict a case in which income effects dominate, so that $p_R^R(p_X)$ is upward-sloping, and the right panels of Figure 3 depict a case in which input cost effects dominate, so that $p_R^R(p_X)$ is downward-sloping. In the latter case, tâtonnement-stability requires that $p_R^R(p_X)$ is flatter than $p_R^X(p_X)$.[16] The intersection of $p_R^R(p_X)$ and $p_R^X(p_X)$ defines the point at which excess demands for X and R are each zero, and thus defines equilibrium p_R and p_X.

Now consider a marginal improvement in A_k, for some sector k. This improvement has two effects. First, from equation (5), a direct effect shifts demand for R_k outward if and only if $\sigma_k > 1$. An outward shift in demand for R_k corresponds to an upward shift in $p_R^R(p_X)$, which increases equilibrium p_R and thus equilibrium R. The increases in equilibrium p_R and R are especially large when $p_R^X(p_X)$ is steep. The top panels of Figure 3 depict a case with $\sigma_k < 1$, and the bottom panels depict a case with $\sigma_k > 1$. This first shift is labeled 1. It corresponds to the partial equilibrium effect of an improvement in efficiency.

Second, raising A_k acts like reducing the cost of inputs in sector k. At any given consumption level c_k, the zero-profit condition in equation (3) requires that the output price falls. The fall in the output price reduces demand for the inputs X_k and R_k, but it also leads consumers to substitute towards good k, which increases demand for X_k and R_k. Because $\epsilon > \sigma_k$, the latter effect dominates. As a result, this output price channel shifts $p_R^R(p_X)$ up and shifts $p_R^X(p_X)$ to the right. These shifts are labeled 2 and 3, respectively, in Figure 3.

[15] Note that $\sigma_i \geq \tilde{\sigma}_i$ if $\sigma_i \geq \epsilon - 1$. Thus the sufficient condition in part (i) of Proposition 1 for a tâtonnement-stable equilibrium ensures that income effects do not dominate in demand for X but do dominate in determining how increasing p_X shifts demand for R.

[16] To see this, note that $\frac{\partial D^R}{\partial p_R}\frac{\partial D^X}{\partial p_X} - \frac{\partial D^R}{\partial p_X}\frac{\partial D^X}{\partial p_R} > 0$ is equivalent to $[\frac{\partial D^X}{\partial p_X}]/[\frac{\partial D^X}{\partial p_R}] > [\frac{\partial D^R}{\partial p_X}]/[\frac{\partial D^R}{\partial p_R}]$, where we recognize that both derivatives with respect to p_R are negative. By the implicit function theorem, this inequality is equivalent to $\mathrm{d}p_R^X(p_X)/\mathrm{d}p_X < \mathrm{d}p_R^R(p_X)/\mathrm{d}p_X$.



(a) $\sigma_k < 1$, strong income effects

(b) $\sigma_k < 1$, weak income effects

(c) $\sigma_k > 1$, strong income effects

(d) $\sigma_k > 1$, weak income effects

Figure 3: How an improvement in A_k affects equilibrium resource use, through the equilibrium p_R (which is at the intersection of p_R^R and p_R^X).

The upward shift in $p_R^R(p_X)$ (labeled 2) always works to increase equilibrium p_R and thus to increase equilibrium R. This is a first general equilibrium channel. It always acts to amplify partial equilibrium rebound. This first general equilibrium channel is especially strong when the shift is large (i.e., when $\epsilon - \sigma_k$ is large) and when $p_R^X(p_X)$ is steep (i.e., when small increases in p_R lead to big reductions in aggregate demand for X).

The rightward shift in $p_R^X(p_R)$ (labeled 3) works to increase equilibrium p_R and R when $p_R^R(p_X)$ is upward-sloping due to strong income effects (left panels) but works to decrease equilibrium p_R and R when $p_R^R(p_X)$ is downward-sloping due to input cost effects (right panels). This is a second general equilibrium channel. It amplifies rebound if $\sigma_i > \tilde{\sigma}_i$ for all sectors i and dampens rebound if $\sigma_i < \tilde{\sigma}_i$ for all sectors i. Intuitively, improving A_k increases demand for X_k as households substitute towards good k. The price p_X must rise to restore equilibrium, which makes other goods $j \neq k$ more expensive (increasing p_j) and thus reduces demand for their factor inputs. However, an increase in p_X also increases household income, which increases the quantity of R_j demanded at any price p_j. As σ_j grows, the increase in p_j reduces demand for R_j to a smaller degree, with the income effect eventually dominating the input cost effect for $\sigma_j > \tilde{\sigma}_j$.

In sum, for $\sigma_i > \tilde{\sigma}_i$ (left panels), both general equilibrium channels work to increase rebound: when the price of c_k falls, the increased demand for X creates income effects that amplify the direct effects of consumer substitution towards good k.[17] However, when $\sigma_i < \tilde{\sigma}_i$ (right panels), the two general equilibrium channels conflict: when the price of c_k falls, the increased demand for X makes production more expensive and thus could potentially reduce aggregate demand for R. We now formally analyze the net effects on equilibrium consumption of R, beginning with the special case in which every sector has the same production technology.

4.2 Symmetric Sectors

We here consider the case in which each sector is initially identical, with $\sigma_i = \sigma_j = \sigma$ and $A_i = A_j = A$ for all $i, j \in \{1, ..., N\}$. Because sectors have identical technologies and

[17] This general equilibrium story bears similarities to the story originally told by Jevons (1865, p. 141):

> Now, if the quantity of coal used in a blast-furnace, for instance, be diminished in comparison with the yield, the profits of the trade will increase, new capital will be attracted, the price of pig-iron will fall, but the demand for it increase; and eventually the greater number of furnaces will more than make up for the diminished consumption of each.

His increasing "profits of the trade" describes the reductions in input costs, his "new capital" reflects how new entrants induce a decline in p_k and thereby restore the zero-profit condition, and his increase in demand for pig-iron reflects increased demand for factor inputs in sector k. As sector k expands (with a "greater number of furnaces"), it can increase overall use of raw resources (such as pig-iron), even though a partial equilibrium analysis would predict that a small elasticity of substitution in sector k would work to reduce resource use (i.e., would emphasize "the diminished consumption of each" furnace). Jevons' story corresponds to the top-left panel of Figure 3.

production functions, they each initially use identical quantities of energy and non-energy inputs.

Consider a 1% improvement in A_k. The appendix shows that

$$A_k \frac{\mathrm{d}R}{\mathrm{d}A_k} = \psi R_k \frac{\overbrace{-(\sigma-1)\left[(1-\sigma)-(\epsilon-\sigma)\alpha_X\right]}^{\text{PE component}} + \overbrace{(\epsilon-\sigma)\,\alpha_R\,\sigma}^{\text{GE component}}}{-(\psi+\sigma)\left[(1-\sigma)-(\epsilon-\sigma)\alpha_X\right] + (\epsilon-\sigma)\alpha_R\,\sigma}. \tag{7}$$

The requirement that $\sigma > \sigma^L$ ensures that the terms in square brackets are negative, which in turn ensures that the denominator is positive. The partial equilibrium (PE) component has the same sign as in our earlier analysis.[18] The general equilibrium (GE) component is novel. We immediately have the following result:

Proposition 2. *The general equilibrium component increases rebound when each sector is initially identical.*

Proof. Follows from (7) and $\sigma < \epsilon$. □

In particular, we see that the GE component increases rebound by an especially large amount when σ is intermediate between 0 and ϵ and the value share α_R of resources is large, which happens when A is large (small) with σ greater (less) than 1. The GE component disappears as σ goes to either zero or ϵ, and also disappears as α_R goes to zero. Intuitively, when the value share of resources is large, an improvement in sector k's energy conversion technology strongly reduces the price of consumption good k, and we saw in Section 4.1 how $\epsilon - \sigma$ determines the sensitivity of demand for X_k and R_k to changes in p_k. The graphical analysis in Section 4.1 showed that at least one of the two outward shifts in demand works to increase the resource price p_R, but the other outward shift can work to decrease p_R. We here see that, in this special case with symmetrical elasticities of substitution and initial technologies, the net effect of these two outward shifts in factor demand is always positive and is proportional to σ. As $\alpha_R \to 0$, the output price p_k does not fall by much following an improvement in A_k; as $\sigma \to \epsilon$, factor demand does not shift out by much as p_k falls; and as $\sigma \to 0$, the conflicting effects of the outward shifts in demand for each factor cancel.

The following proposition establishes when the improvement in efficiency increases total resource use:

Proposition 3. *There exists $x < \alpha_X$ such that improving A_k increases R if $\sigma \geq x$. Alternately, improving A_k increases R if $\epsilon > \max\{2-\alpha_X, 1/\alpha_X\}$.*

[18] Using L'Hôpital's rule, we have that as $\psi \to \infty$, the partial equilibrium component (including the denominator) becomes $R_k\,(\sigma-1)$. Therefore, when resource supply is perfectly elastic, the partial equilibrium component is exactly as in Section 3. Perfectly elastic resource supply is consistent with the partial equilibrium assumption of a fixed resource price.

Proof. Rearranging, the right-hand side of equation (7) becomes $(1-\sigma)^2 + (\epsilon - \sigma)[\sigma - \alpha_X]$. This is positive for $\sigma \geq \alpha_X$ and for σ not too much smaller than α_X, which establishes the first part of the proposition. Expanding, this expression becomes $\sigma[\epsilon - 2 + \alpha_X] + [1 - \epsilon\,\alpha_X]$. $\epsilon > 2 - \alpha_X$ ensures that the first term in brackets is positive, and $\epsilon > 1/\alpha_X$ ensures that the second term in brackets is positive. Further, substituting $\alpha_R = 1 - \alpha_X$ into the definition of σ_L, we have $\sigma_L = (1 - \alpha_X\,\epsilon)/(1 - \alpha_X)$. Thus, $\epsilon > 1/\alpha_X$ implies that $\sigma^L < 0$, which implies a stable equilibrium for all $\sigma > 0$ by Proposition 1. This establishes the second part of the proposition. □

In the partial equilibrium analysis, backfire arose if and only if $\sigma > 1$, and much literature has argued that backfire is unlikely because $\sigma < 1$. Indeed, the partial equilibrium component in equation (7) produces backfire if and only if $\sigma > 1$. However, noting that $\alpha_X < 1$, we now see that accounting for general equilibrium effects makes backfire occur for a strictly larger set of σ, and backfire can occur even for very small σ if either ϵ is large or energy is so abundant that α_X is large. The magnitude of σ is not a reliable guide to the likelihood of backfire in a general equilibrium setting.

The next proposition establishes that rebound is always positive. Define

$$Savings^{GE} \triangleq -A_k \frac{\mathrm{d}R}{\mathrm{d}A_k}, \quad Rebound^{GE} \triangleq \frac{Savings^{GE} - Savings^{eng}}{Savings^{eng}}.$$

Then we have:

Proposition 4. *$Rebound^{GE} > 0$.*

Proof. The derivative of $A_k \frac{\mathrm{d}R}{\mathrm{d}A_k}$ with respect to ψ is:

$$A_k \frac{\mathrm{d}R}{\mathrm{d}A_k} \frac{1}{\psi} \left\{ 1 - \frac{-\psi\,[(1-\sigma) - (\epsilon-\sigma)\alpha_X]}{-(\psi+\sigma)\,[(1-\sigma) - (\epsilon-\sigma)\alpha_X] + (\epsilon-\sigma)\alpha_R\,\sigma} \right\}.$$

The fraction in curly braces is strictly less than 1. Therefore the term in curly braces is positive and the expression has the same sign as $\mathrm{d}R/\mathrm{d}A_k$. Therefore $A_k \mathrm{d}R/\mathrm{d}A_k$ is decreasing in ψ whenever $\mathrm{d}R/\mathrm{d}A_k < 0$. As $\psi \to \infty$, we have (using L'Hôpital's rule):

$$\begin{aligned} A_k \frac{\mathrm{d}R}{\mathrm{d}A_k} \to & R_k - \sigma\, R_k \left\{ 1 - \frac{(\epsilon-\sigma)\,\alpha_R}{-\,[(1-\sigma) - (\epsilon-\sigma)\alpha_X]} \right\} \\ = & R_k - \sigma\, R_k \left\{ 1 - \frac{(\epsilon-\sigma) - (\epsilon-\sigma)\,\alpha_X}{-\,[(1-\sigma) - (\epsilon-\sigma)\alpha_X]} \right\}. \end{aligned}$$

Recall that $\sigma > \sigma^L$ implies that the fraction's denominator is positive. Note that the fraction's numerator is positive because $\alpha_X \leq 1$. $\epsilon > 1$ implies that the fraction is strictly greater than 1, which means that the term in curly braces is negative and we have $Rebound^{GE} > 0$

as $\psi \to \infty$. And because $\mathrm{d}R/\mathrm{d}A_k < 0$ is both necessary for $Rebound^{GE} < 0$ and implies that $\mathrm{d}R/\mathrm{d}A_k$ is decreasing in ψ (and thus is negative for all larger ψ when negative at a given ψ), we have that $Rebound^{GE} > 0$ more generally.

□

If economic responses increase energy resource savings beyond the no-rebound or "engineering" calculation, then we have negative rebound, known as "super-conservation." We here see that economic responses always undercut "engineering" savings in this general equilibrium setting. Contrary to claims in the literature (Turner, 2009; Wei, 2010), negative rebound or "super-conservation" cannot occur.[19]

Finally, consider how increasing the number of sectors in the economy changes the general equilibrium component's share of rebound.

Proposition 5. *The general equilibrium component's relative contribution to rebound increases in N if and only if $\sigma > 1$*

Proof. Using $\alpha_R = 1 - \alpha_X$, the ratio of the general equilibrium component to the partial equilibrium component is $\frac{\epsilon-\sigma}{1-\sigma} \frac{\sigma\,\alpha_R}{-(\epsilon-1)+(\epsilon-\sigma)\alpha_R}$. The denominator of the right-hand fraction is negative by $\sigma > \sigma^L$. Changing N affects only α_R. The magnitude of the numerator increases in α_R, and the magnitude of the denominator decreases in α_R. The relative contribution of the general equilibrium component to rebound therefore increases in α_R. Note that $X_k = L/N$ and $R_k = R/N$. As N increases, X_k/R_k falls because L is fixed while R is free to increase (because $\psi > 0$). A decline in X_k/R_k increases α_R if and only if $\sigma > 1$, as can be seen by using the firm's first-order conditions (equations (1) and (2)). The proposition follows. □

Intuitively, the general equilibrium component becomes important when the value share of resources is large because the improvement in the efficiency of energy production then affects the price of consumption good k to an especially strong degree. Increasing N makes the non-energy input relatively scarcer because its supply is fixed independently of N, and making the non-energy input relatively scarcer increases the value share of resources if and only if $\sigma > 1$. Thus, when $\sigma > 1$, the general equilibrium component becomes relatively more important as sector k becomes a smaller part of the overall economy.[20]

[19] Turner (2009) attributes her computable general equilibrium model's cases with negative rebound to "disinvestment" in the energy supply sectors. Wei (2010) is not clear about what drives negative rebound in his analytic setting. Intuitively, negative rebound would seem plausible if supplying additional energy resources stimulated demand for resources or if the utility (or production) function were non-homothetic, with resource demand decreasing in income (or in consumption good production).

[20] We would obtain the opposite result if our thought experiment were to increase L along with N so that L/N is held constant. Then R would become relatively scarce as we increase N, assuming that resource supply is not perfectly elastic ($\psi < \infty$).

4.3 Heterogeneous Sectors

We now return to the general case in which σ_i and A_i can vary by sector i. Again consider a 1% improvement in A_k. The appendix shows that:

$$A_k \frac{\mathrm{d}R}{\mathrm{d}A_k} \propto \overbrace{-(\sigma_k - 1) R_k \sum_{i=1}^{N} X_i \left[(1-\sigma_i) - (\epsilon - \sigma_i)\alpha_{Xi}\right]}^{\text{PE component}} \\ \underbrace{+ (\epsilon - \sigma_k)\alpha_{Rk} \left\{ R_k \sum_{i=1}^{N} X_i \, \sigma_i + \sum_{i=1}^{N} R_k R_i \left[\frac{X_k}{R_k} - \frac{X_i}{R_i}\right] \left[1 - (\epsilon - \sigma_i)\alpha_{Xi}\right] \right\}}_{\text{GE component}}, \tag{8}$$

where the factored terms are independent of k. As in the case of symmetric sectors, the requirement that $\sigma_i > \sigma_i^L$ ensures that the term in square brackets in the partial equilibrium component is negative. Therefore we have that the partial equilibrium component is positive (promoting backfire) if and only if $\sigma_k > 1$, a condition that is by now familiar. And as in the case with symmetric sectors, we see that the general equilibrium component vanishes as either $\sigma_k \to \epsilon$ or as the value share of resources in sector k goes to 0.

However, in contrast to the case with symmetric sectors, the sign of the general equilibrium component is here ambiguous. The general equilibrium component derives from the reduction in p_k due to improved efficiency and the zero-profit condition. The reduction in p_k depends on the value share of resources in sector k (α_{Rk}), and its impact on factor demand in sector k scales with $\epsilon - \sigma_k$, from equations (4) and (5). As described previously, households substitute towards sector k, which tends to increase its production c_k.

The first term in curly braces is always positive (promoting backfire) and is large when the elasticity of substitution between energy and non-energy inputs is large in many sectors. This term reflects how the efficiency-induced reduction in p_k increases demand for R_k. Firms in sector k can increase their resource use most strongly when they can access additional quantities of the labor-capital aggregate. A small increase in p_X frees up a lot of the labor-capital aggregate X when sectors with high use of X also have a high elasticity of substitution. In this case, $\sum_i \sigma_i X_i$ is large, and sector k can strongly increase its quantity of resource use to satisfy household demand. In terms of the graphical analysis of Section 4.1, increasing this first term in curly braces makes $p_R^X(p_X)$ steeper, which increases rebound from the outward shift in $p_R^R(p_X)$. This first term in curly braces is the only general equilibrium term that survived in the case with symmetric sectors.

The second term in curly braces captures income effects and has an ambiguous sign. It reflects how the efficiency-induced reduction in p_k changes p_X (first set of square brackets) and thus changes resource use (second set of square brackets).[21] The reduction in p_k works to

[21]Referring to the graphical analysis of Section 4.1, the first pair of square brackets controls the relative

increase demand for both X_k and R_k. When sector k is especially X-intensive, the reduction in p_k tends to increase aggregate demand for X and thus raise p_X, corresponding to the first set of square brackets being positive. As described in Section 4.1, the second set of square brackets determines whether the income effects of greater p_X dominate the input cost effects of greater p_X. Income effects make resource consumption increase in p_X, but input cost effects make resource consumption decrease in p_X. When sector k is especially X-intensive (especially resource-intensive), then p_X increases (falls), and this change increases resource use when income (input cost) effects dominate. This second term in curly braces vanishes in the case of symmetric sectors, because all sectors are then equally X-intensive.

The next proposition establishes a sufficient condition for an improvement in the energy efficiency of sector k to reduce total resource use:

Proposition 6. *If $\epsilon \leq 2$ and each R_i/X_i is sufficiently large, then there exists $\hat{\sigma}_i \in (\sigma_i^L, 1)$ such that improving A_k reduces R if $\sigma_i \in (\sigma_i^L, \hat{\sigma}_i]$ for all $i \in \{1, ..., N\}$. If $\epsilon > 2$ and each R_i/X_i is sufficiently large, then there exist $\hat{\sigma}_k \in (\sigma_k^L, 1)$ and $\hat{\sigma}_i > \hat{\sigma}_k$ such that improving A_k reduces R if $\sigma_k \leq \hat{\sigma}_k$ and $\sigma_i \leq \hat{\sigma}_i$ for all $i \in \{1, ..., N\}$.*

Proof. See appendix. □

We see that it is possible for improving the efficiency of energy conversion technology to reduce resource use. In particular, if resources are abundant relative to the non-energy input and the elasticity of substitution between energy and non-energy inputs is sufficiently smaller than 1, then an improvement in energy efficiency will reduce total resource use. Connecting to the graphical analysis, the condition on σ_i ensures, first, that the partial equilibrium effect is sufficiently small that improving A_k directly shifts $p_R^R(p_X)$ downward and, second, that income effects are sufficiently weak to make $p_R^R(p_X)$ downward-sloping.

The next proposition establishes that it is also possible for efficiency policies to backfire, increasing total resource consumption:

Proposition 7. *An improvement in A_k increases R if either of the following hold:*

i $\epsilon \geq 2$ with $\sigma_i \geq \epsilon - \frac{1}{\alpha_{Xi}}$ for all $i \in \{1, ..., N\}$, or

ii $\epsilon \leq 2$ with $\sigma_i \geq \epsilon - \frac{\epsilon-1}{\alpha_{Xi}}$ for all $i \in \{1, ..., N\}$, where $\epsilon - \frac{\epsilon-1}{\alpha_{Xi}} < 1$.

Proof. See appendix. □

Recognizing that the value shares are less than 1, we have the following corollary:

Corollary 8. *An improvement in A_k increases R if $\sigma_i \geq \max\{1, \epsilon - 1\}$ for all $i \in \{1, ..., N\}$.*

size of the shifts in $p_R^R(p_X)$ and $p_R^X(p_X)$, and the second pair of square brackets controls both whether $p_R^R(p_X)$ slopes up (when positive) or down (when negative) and the slope of $p_R^X(p_X)$ (with it being steeper when the term in brackets is negative).

Proof. Follows from Proposition 7 and the observation that $\epsilon - \frac{1}{\alpha_{Xi}} \leq \epsilon - 1$ and $\epsilon - \frac{\epsilon-1}{\alpha_{Xi}} \leq 1$. □

We see that σ_i sufficiently large guarantees backfire in this general equilibrium setting, as in the partial equilibrium setting. Note that $\epsilon - \frac{1}{\alpha_{Xi}}$ defined $\tilde{\sigma}_i$ in the graphical analysis. The first sufficient condition in Proposition 7 therefore ensures that income effects are sufficiently strong that $p_R^R(p_X)$ is upward-sloping in the graphical analysis of Section 4.1, in which case we saw that all general equilibrium channels amplify rebound.

Now consider which sectors generate positive rebound through general equilibrium channels, for a special case in which the elasticity of substitution is the same in each sector:

Proposition 9. *Fix $\sigma_i = \sigma_j = \sigma$ for all i, j.*

i Assume that $\sigma \geq \epsilon - \frac{1}{\alpha_{Xi}}$, which holds if $\sigma \geq \epsilon - 1$. Targeting the most (least) efficient sector increases rebound through general equilibrium channels if $\sigma <$ ($>$) 1.

ii Assume that $\sigma \leq \epsilon - \frac{1}{\alpha_{Xi}}$. Targeting the most (least) efficient sector increases rebound through general equilibrium channels if $\sigma >$ ($<$) 1.

Proof. See appendix. □

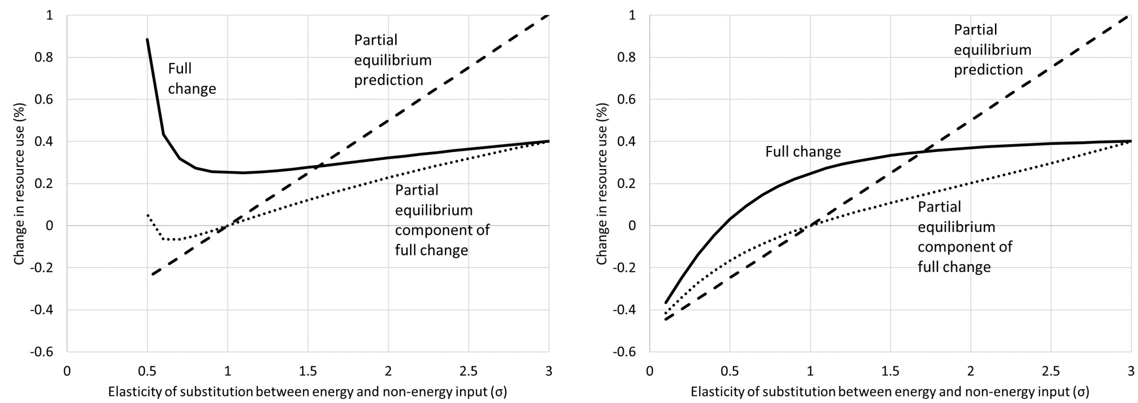
We see that the sign of general equilibrium effects depends, first, on whether the improvement in efficiency occurs in a relatively efficient sector or in a relatively inefficient sector and, second, on the magnitudes of σ and ϵ. If σ is large relative to ϵ and greater than 1, or if σ is small relative to ϵ and less than 1, then general equilibrium channels increase rebound when the targeted sector was relatively inefficient. However, if σ is of intermediate magnitude, then general equilibrium effects tend to increase rebound when the targeted sector was relatively efficient.[22]

5 Numerical Examples

We now consider numerical examples that illustrate the effects described above and further explore how to target an efficiency policy so as to achieve the greatest reduction in resource use.[23]

[22] Recall again that $\epsilon - \frac{1}{\alpha_{Xi}}$ defined $\tilde{\sigma}_i$ in the graphical analysis. The assumption that $\sigma > \tilde{\sigma}_i$ ensures that $p_R^R(p_X)$ is upward-sloping. In this case, the first term in square brackets is positive and greater p_X works to increase resource use through strong income effects. Improving A_k increases p_X when sector k is especially X-intensive. And sector k is especially X-intensive when its energy conversion technology is relatively advanced and $\sigma < 1$, and also when its technology is relatively backward and $\sigma > 1$. In contrast, when $\sigma < \tilde{\sigma}_i$, the input cost effects of greater p_X dominate the income effects, and this logic reverses.

[23] All of the simulations use $\kappa = 0.5$, $\epsilon = 3$, $L = 1$, $\Psi = 1$, and $\psi = 2$, and all simulations study a 1% improvement in some one sector's energy conversion technology. We solve for the equilibrium by seeking the p_R and p_X that set $D^R(p_R, p_X)$ and $D^X(p_R, p_X)$ to zero. All plotted cases satisfy the conditions for stability. Unstable equilibria arose only in cases with symmetric sectors: in particular, when $A = 0.75$ and $\sigma \leq 0.4$, and when $\sigma = 0.5$ and $A \leq 0.7$.

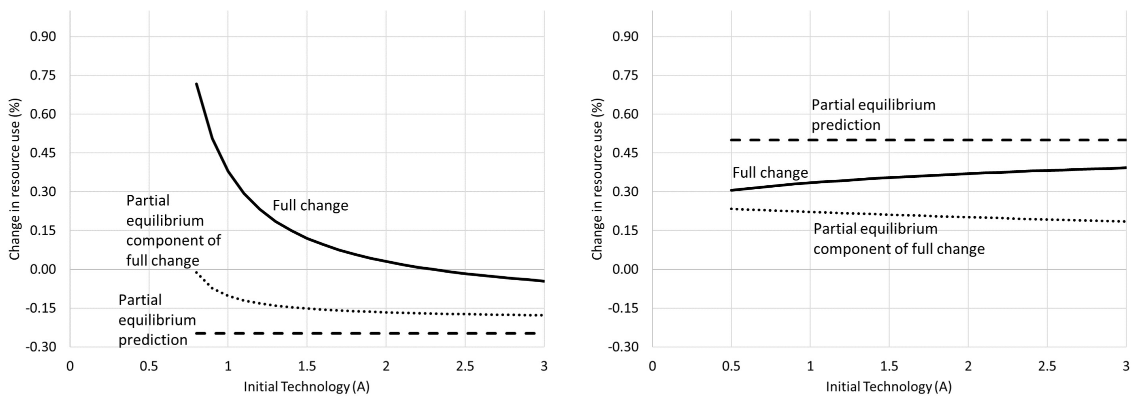


(a) $A = 0.75$ (b) $A = 2$

Figure 4: The change in resource use due to an efficiency policy, as a function of the elasticity of substitution σ. Both sectors ($N = 2$) begin with the same level of technology (left: $A = 0.75$; right: $A = 2$) and have the same elasticity of substitution.

Begin by considering how the elasticity of substitution between energy and non-energy inputs (σ) determines the change in resource use resulting from an improvement in the quality of energy conversion technology. Figure 4 plots this change in a two-sector model ($N = 2$) that gives each sector the same elasticity of substitution σ and the same initial quality of technology, either $A = 0.75$ (left) or $A = 2$ (right). The solid line shows the total change in resource use resulting from improving one sector's technology, and the dashed line shows the partial equilibrium change in resource use, as analyzed in Section 3. Consistent with the theoretical analysis, we see that a partial equilibrium analysis predicts that an improvement in efficiency increases resource use whenever $\sigma > 1$ and reduces resource use otherwise. However, we see that general equilibrium effects mean that the actual change in resource use can be positive for σ much smaller than 1. Ignoring general equilibrium effects can lead analysts to incorrectly predict savings from efficiency policies when the elasticity of substitution is small. The dotted line plots the partial equilibrium component from equation (7), so that the gap between the solid and dotted lines is the general equilibrium component. We see that the general equilibrium component is always positive and can be much larger than the partial equilibrium component.

Now consider how the initial level of technology matters for the consequences of an improvement in energy efficiency. Figure 5 fixes the elasticity of substitution at either $\sigma = 0.5$ (left) or $\sigma = 2$ (right) and varies the quality of technology A that is common to both sectors. In either case, the partial equilibrium predictions are not sensitive to the initial quality of technology. In contrast, we see that the initial quality of technology matters for the full change in resource use. The general equilibrium component (the gap between the solid and

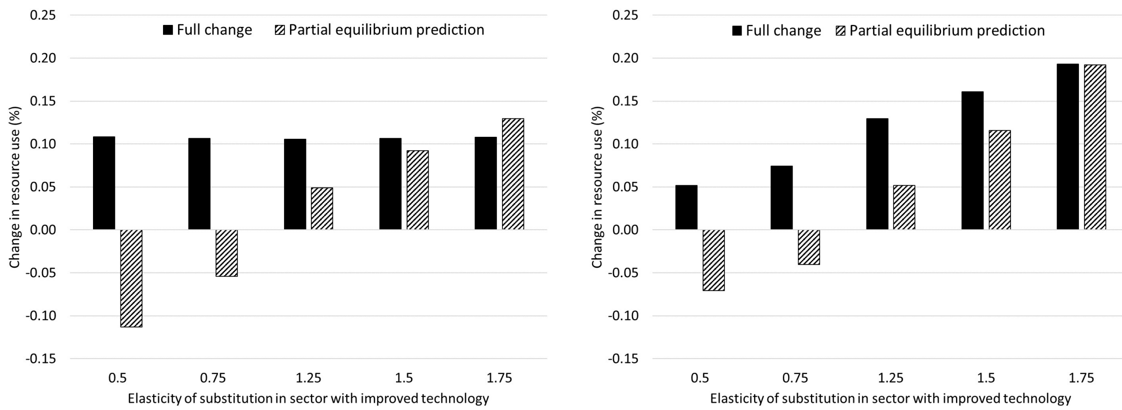


(a) $\sigma = 0.5$ (b) $\sigma = 2$

Figure 5: The total change in resource use due to an efficiency policy, as a function of the initial level of technology A. Both sectors ($N = 2$) begin with the same level of technology and have the same elasticity of substitution (left: $\sigma = 0.5$; right: $\sigma = 2$).

dotted lines) is again always positive, and it grows in A for $\sigma = 2$ but shrinks in A for $\sigma = 0.5$. This pattern arises because the value share of resources increases in A if and only if $\sigma > 1$, and we have seen that the general equilibrium component is proportional to the value share of resources. When that value share becomes large (e.g., when we have both small A and small σ), the general equilibrium component can be many times the size of the partial equilibrium component.

We have thus far analyzed changes in resource use in an economy with initially identical sectors, but actual economies have heterogeneous sectors. A natural question is which sectors policymakers should target with an efficiency policy in order to reduce resource use, and which sectors policymakers should avoid targeting with an efficiency policy. To study these questions, we now consider a setting with five sectors ($N = 5$). In Figure 6, each sector has common technology ($A_i = 0.75$ in the left panel or $A_i = 2$ in the right panel, for $i \in \{1, ..., 5\}$), but the elasticity of substitution σ_i varies between sectors, taking on values of 0.5, 0.75, 1.25, 1.50, and 1.75. The solid bars give the full change in resource use resulting from a 1% improvement in the efficiency of a given sector, and the hatched bars give the change in resource use predicted by a partial equilibrium analysis of the targeted sector. As should be expected, these partial equilibrium changes are negative for sectors with $\sigma_i < 1$ and positive for sectors with $\sigma_i > 1$. The full change in resource use is positive in every sector and often much larger than the change predicted by the partial equilibrium setting (even when the signs match). A policymaker cannot avoid inducing backfire in this example, but targeting the sector with the smallest elasticity of substitution can mitigate the magnitude of backfire when sectors are fairly energy efficient to begin with (right panel). The partial



(a) $A = 0.75$ (b) $A = 2$

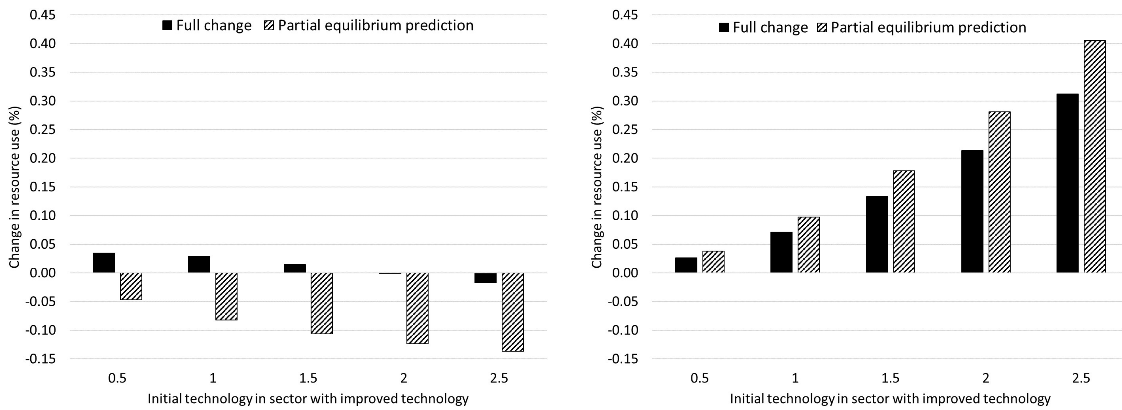
Figure 6: Each of five sectors ($N = 5$) has the same initial level of technology (left: $A = 0.75$; right: $A = 2$) but a different elasticity of substitution σ_i. The plotted changes in resource extraction vary which sector receives a 1% improvement in its energy conversion technology.

equilibrium and general equilibrium analyses each suggest targeting the sector with the smallest σ_i, but the partial equilibrium analysis suggests much greater gains from targeting the proper sector than does the general equilibrium analysis.

Figure 7 undertakes a similar thought experiment, but now fixes the elasticity of substitution in each sector at either $\sigma = 0.5$ (left) or $\sigma = 2$ (right) and allows the initial quality of technology A_i to vary across sectors, taking on values of 0.5, 1, 1.5, 2, and 2.5. We see that the partial equilibrium analysis would suggest targeting the most efficient sector when $\sigma = 0.5$ (in order to achieve the greatest savings in resource use) but would suggest targeting the least efficient sector when $\sigma = 2$ (in order to minimize backfire). The general equilibrium analysis makes qualitatively similar recommendations, but suggests much smaller gains from properly targeting the policy. Thus, the partial equilibrium analysis again can serve as a good guide to choosing which sector to target with an efficiency policy, but the partial equilibrium analysis again can produce highly misleading estimates of the net benefits of such a policy.

6 Conclusions

In order to matter for climate change, policies to increase energy efficiency must be large-scale. Such big policies are likely to have general equilibrium consequences. We have seen that these general equilibrium consequences are likely to undercut the policies' environmental objectives. Further, these policies typically target sectors in which energy use is perceived to



(a) $\sigma = 0.5$ (b) $\sigma = 2$

Figure 7: Each of five sectors ($N = 5$) has the same elasticity of substitution (left: $\sigma = 0.5$; right: $\sigma = 2$) but a different initial quality of technology A_i. The plotted changes in resource extraction vary which sector receives a 1% improvement in its energy conversion technology.

be important. Such sectors are likely to be ones in which the value share of energy resources is especially high, and we have seen that these are precisely the sectors in which general equilibrium channels will be especially important. This conclusion should introduce additional pessimism about the potential for energy efficiency policies to address climate change. Future work should consider interactions with innovation, with economic growth, and with international trade, and future work should determine which of the analyzed channels drives the large rebound estimates reported by computable general equilibrium models.

First Appendix: Partial equilibrium analysis with energy services as a direct input to utility

We here provide a partial equilibrium analysis of a representative setting in which energy services are a direct input to utility (e.g., Chan and Gillingham, 2015). Let household utility be $u(A\,R, X)$ with $A > 0$ the quality of energy technology, R the chosen consumption of energy, and X the chosen consumption of non-energy goods. The utility function is increasing and strictly concave. The household's budget constraint is $p_R R + p_X X \leq w$, with $w > 0$ given. From this budget constraint, we have $X = [w - p_R R]/p_X$. The household then solves

$$\max_R \; u\left(A\,R, \frac{w - p_R R}{p_X}\right).$$

The first-order condition for a maximum is

$$A\,u_1 - \frac{p_R}{p_X} u_2 = 0,$$

where subscript i on u indicates a partial derivative with respect to the ith argument. The second-order condition for a global maximum is

$$A^2 u_{11} - 2A\frac{p_R}{p_X} u_{12} + \left(\frac{p_R}{p_X}\right)^2 u_{22} < 0.$$

This holds as long as

$$u_{12} > \frac{A^2 u_{11} + \left(\frac{p_R}{p_X}\right)^2 u_{22}}{2A\frac{p_R}{p_X}} = \frac{1}{2}\frac{p_X A}{p_R} u_{11} + \frac{1}{2}\frac{p_R}{p_X A} u_{22}.$$

The right-hand side is negative. We assume that the second-order condition holds.

Applying the implicit function theorem to the first-order condition, we have:

$$\frac{\mathrm{d}R^*}{\mathrm{d}A} = -\frac{u_1 + R\,A\,u_{11} - R\frac{p_R}{p_X} u_{12}}{A^2 u_{11} - 2A\frac{p_R}{p_X} u_{12} + \left(\frac{p_R}{p_X}\right)^2 u_{22}}.$$

The denominator is negative when the second-order condition holds. The derivative is therefore negative if and only if

$$u_1 + R\,A\,u_{11} - R\frac{p_R}{p_X} u_{12} \leq 0. \tag{9}$$

The first term is positive and the second is negative. The first two terms combine to be negative if and only if

$$\frac{u_1}{u_{11}}\frac{1}{R\,A} \geq -1.$$

The left-hand side is the elasticity of demand for energy services $R\,A$, going to 0 as $u_{11} \to -\infty$ and going to negative infinity as $u_{11} \to 0$. Thus we have that the first two terms generate backfire if and only if demand for energy services is elastic. This result is the analogue of the main text's result that backfire arises in partial equilibrium if and only if the elasticity of substitution between energy and non-energy inputs is greater than 1.

The new term in equation (9) relative to the main text's partial equilibrium analysis is the term with u_{12}, which works to reduce resource use if and only if $u_{12} > 0$. This term is the analogue to the decline in the price of output from sector k in the main text's general equilibrium analysis, except here holding factor prices and income constant. Improving A increases the quantity of energy services consumed. When $u_{12} > 0$, the two inputs to utility are complements and the efficiency-induced increase in consumption of energy services increases the marginal utility of the non-energy input. The household therefore allocates more of the fixed budget to the non-energy input, which works to reduce resource use. When $u_{12} < 0$, the energy and non-energy inputs are substitutes. This is the analogue of the main text's assumption that $\epsilon > 1$. Now the efficiency-induced increase in energy services leads the household to substitute away from the non-energy input, which works to undercut any resource savings from an efficiency policy. This result validates the claim in footnote 14 in the main text.

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Second Appendix: Proofs and Derivations

Proof of Proposition 1

Begin by considering the partial derivatives of D^R and D^X with respect to p_R and p_X, respectively:

$$\begin{aligned}
\frac{\partial D^R(p_R, p_X)}{\partial p_R} =& \sum_{i=1}^{N} R_i \left[-\psi - \sigma_i + (\sigma_i - \epsilon) R_i p_i^{\epsilon-1} \frac{p_R}{p_X} \right] \frac{1}{p_R} \\
=& \sum_{i=1}^{N} R_i \left[-\psi - \sigma_i - (\epsilon - \sigma_i) \alpha_{Ri} \right] \frac{1}{p_R} \\
<& 0, \\
\frac{\partial D^X(p_R, p_X)}{\partial p_X} =& \sum_{i=1}^{N} X_i \left[(1 - \sigma_i) + (\sigma_i - \epsilon) X_i p_i^{\epsilon-1} \right] \frac{1}{p_X} \\
=& \sum_{i=1}^{N} X_i \left[(1 - \sigma_i) - (\epsilon - \sigma_i) \alpha_{Xi} \right] \frac{1}{p_X} \\
<& 0 \text{ if, } \forall i \in \{1, ..., N\},\ \sigma_i > \frac{1 - \alpha_{Xi}\epsilon}{1 - \alpha_{Xi}} = \epsilon - \frac{\epsilon - 1}{\alpha_{Ri}} \triangleq \sigma_i^L.
\end{aligned}$$

In the partial derivative of D^R, we use $\Psi p_R^{\psi} = R$. We see that assuming $\sigma_i > \sigma_i^L$ ensures that $\frac{\partial D^R(p_R, p_X)}{\partial p_R} + \frac{\partial D^X(p_R, p_X)}{\partial p_X} < 0$, which was the first condition required for stability.

Now consider the other condition required for stability. Differentiating, rearranging, and using $\alpha_{Ri} + \alpha_{Xi} = 1$ yields:

$$\begin{aligned}
&\frac{\partial D^R(p_R,p_X)}{\partial p_R}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial p_R} \\
&\quad = \left\{\sum_{i=1}^N R_i\left[(-\psi-\sigma_i)-(\epsilon-\sigma_i)\alpha_{Ri}\right]\frac{1}{p_R}\right\}\left\{\sum_{i=1}^N X_i\left[(1-\sigma_i)-(\epsilon-\sigma_i)\alpha_{Xi}\right]\frac{1}{p_X}\right\} \\
&\qquad + \left\{\sum_{i=1}^N R_i\left[1-(\epsilon-\sigma_i)\alpha_{Xi}\right]\frac{1}{p_X}\right\}\left\{\sum_{i=1}^N X_i(\epsilon-\sigma_i)\alpha_{Ri}\frac{1}{p_R}\right\} \\
&\quad = -\frac{1}{p_R\,p_X}\left\{\sum_{i=1}^N R_i\,(\psi+\sigma_i)\right\}\left\{\sum_{i=1}^N X_i\left[(1-\sigma_i)-(\epsilon-\sigma_i)\alpha_{Xi}\right]\right\} \\
&\qquad - \frac{1}{p_R\,p_X}\left\{\sum_{i=1}^N R_i(\epsilon-\sigma_i)\alpha_{Ri}\right\}\left\{\sum_{i=1}^N X_i[(1-\sigma_i)-(\epsilon-\sigma_i)]\right\} \\
&\qquad + \frac{1}{p_R\,p_X}\left\{\sum_{i=1}^N R_i[1-(\epsilon-\sigma_i)]\right\}\left\{\sum_{i=1}^N X_i(\epsilon-\sigma_i)\alpha_{Ri}\right\}.
\end{aligned}$$

The first two lines on the right-hand side of the final equality are positive if $\sigma_i > \sigma_i^L$. The last line is positive if $\sigma_i \geq \epsilon - 1$ for all $i \in \{1, ..., N\}$. This establishes the first sufficient condition in the proposition.

Define $W_i \triangleq 1-(\epsilon-\sigma_i)$ and $Z_i \triangleq (\epsilon-\sigma_i)\,\alpha_{Ri}$. Consider the case in which each $\sigma_i < \epsilon-1$, which implies that each $W_i < 0$. Let $\mathbf{R}$, $\mathbf{X}$, $\mathbf{W}$, and $\mathbf{Z}$ be the $N \times 1$ column vectors of R_i, X_i, W_i, and Z_i, respectively. Rewrite the last expression as:

$$\begin{aligned}
&\frac{\partial D^R(p_R,p_X)}{\partial p_R}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial p_R} \\
&\quad = -\frac{1}{p_R\,p_X}\left\{\sum_{i=1}^N R_i\,(\psi+\sigma_i)\right\}\left\{\sum_{i=1}^N X_i\left[(1-\sigma_i)-(\epsilon-\sigma_i)\alpha_{Xi}\right]\right\} \\
&\qquad + \frac{1}{p_R\,p_X}\left\{\sum_{i=1}^N R_i(\epsilon-\sigma_i)\alpha_{Ri}\right\}\left\{\sum_{i=1}^N X_i\,\sigma_i\right\} \\
&\qquad + \frac{1}{p_R\,p_X}\left\{\mathbf{W}'\mathbf{R}\mathbf{Z}'\mathbf{X} - \mathbf{Z}'\mathbf{R}\mathbf{W}'\mathbf{X}\right\}.
\end{aligned}$$

The first line on the right-hand side of the equality is positive if $\sigma_i > \sigma_i^L$, and the second line is always positive. Consider the final line:

$$\begin{aligned}
&0 < \mathbf{W}'\mathbf{R}\mathbf{Z}'\mathbf{X} - \mathbf{Z}'\mathbf{R}\mathbf{W}'\mathbf{X} \\
\Leftrightarrow\quad &\frac{-\mathbf{W}'\mathbf{R}}{-\mathbf{W}'\mathbf{X}} < \frac{\mathbf{Z}'\mathbf{R}}{\mathbf{Z}'\mathbf{X}}.
\end{aligned} \tag{A-1}$$

Now fix $\sigma_i = \sigma_j = \sigma$ for all i, j. W_i becomes independent of i. So inequality (A-1) becomes

$$\frac{R}{L} < \frac{\mathbf{Z}'\mathbf{R}}{\mathbf{Z}'\mathbf{X}}. \tag{A-2}$$

Note that Z_i varies with α_{Ri} and thus with A_i. Using the definition of α and equations (1) and (2), we have:

$$\frac{\alpha_{Ri}}{\alpha_{Xi}} = \frac{1-\kappa}{\kappa}\left(\frac{E_i}{X_i}\right)^{\frac{\sigma_i - 1}{\sigma_i}}.$$

Thus α_{Ri} and Z_i are large in sectors with large A_i if and only if $\sigma > 1$. From equations (3) and (4), we have:

$$\frac{\partial X_i(p_R, p_X; A_i)}{\partial A_i} = X_i(\epsilon - \sigma_i)\alpha_{Ri}\frac{1}{A_i} > 0.$$

X_i and Z_i covary positively (negatively) across sectors i if $\sigma >$ ($<$) 1. And from equations (3) and (5), we have:

$$\frac{\partial R_i(p_R, p_X; A_i)}{\partial A_i} = R_i\left[(\sigma_i - 1) + (\epsilon - \sigma_i)\alpha_{Ri}\right]\frac{1}{A_i} > 0 \text{ iff } \sigma_i > \frac{1-\alpha_{Ri}\epsilon}{1-\alpha_{Ri}} = \epsilon - \frac{\epsilon - 1}{\alpha_{Xi}}.$$

Note that $\epsilon - \frac{\epsilon-1}{\alpha_{Xi}} < 1$. If $\sigma < 1$, then $\mathbf{Z}'\mathbf{X} < \sum_{i=1}^N Z_i \sum_{i=1}^N X_i$, and if $\sigma < \epsilon - \frac{\epsilon-1}{\alpha_{Xi}}$, then $\mathbf{Z}'\mathbf{R} > \sum_{i=1}^N Z_i \sum_{i=1}^N R_i$. Under these conditions, we have:

$$\frac{\mathbf{Z}'\mathbf{R}}{\mathbf{Z}'\mathbf{X}} > \frac{\sum_{i=1}^N Z_i \sum_{i=1}^N R_i}{\sum_{i=1}^N Z_i \sum_{i=1}^N X_i} = \frac{R}{L}.$$

Thus, when $\sigma_i = \sigma_j = \sigma$, $\sigma < \epsilon - 1$, and $\sigma < \epsilon - \frac{\epsilon-1}{\alpha_{Xi}} < 1$, inequality (A-2) holds, which means that inequality (A-1) holds. This establishes the second sufficient condition in the proposition.

Now assume that $\sigma_i = \sigma_j$ and $A_i = A_j$ for all $i, j \in \{1, ..., N\}$. Algebraic manipulations yield:

$$\begin{aligned}&\frac{\partial D^R(p_R, p_X)}{\partial p_R}\frac{\partial D^X(p_R, p_X)}{\partial p_X} - \frac{\partial D^R(p_R, p_X)}{\partial p_X}\frac{\partial D^X(p_R, p_X)}{\partial p_R}\\ &\quad = \frac{LR}{p_R\, p_X}\left\{-(\psi + \sigma)\left[(1-\sigma) - (\epsilon - \sigma)\alpha_X\right] + \sigma(\epsilon - \sigma)\alpha_R\right\} > 0,\end{aligned} \tag{A-3}$$

where we no longer need i subscripts on α. The term in square brackets is positive by our assumption that $\sigma > \sigma^L$. The other term in braces is positive by $\epsilon > \sigma$. We have established the third sufficient condition in the proposition.

Derivation of equation (8)

Using the implicit function theorem on the system defined by $D^R(p_R, p_X)$ and $D^X(p_R, p_X)$ and recognizing that $\mathrm{d}R/\mathrm{d}p_R = \psi R/p_R$, we have that:

$$\frac{\mathrm{d}R}{\mathrm{d}A_k} = \frac{\mathrm{d}R}{\mathrm{d}p_R}\frac{\mathrm{d}p_R}{\mathrm{d}A_k} = -\psi\frac{R}{p_R}\,\frac{\frac{\partial D^R(p_R,p_X)}{\partial A_k}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial A_k}}{\frac{\partial D^R(p_R,p_X)}{\partial p_R}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial p_R}}.$$

Stability implies that the denominator is positive. Factoring the denominator and $\mathrm{d}R/\mathrm{d}p_R$ and differentiating D^R and D^X, we have that:

$$\begin{aligned}
\frac{\mathrm{d}R}{\mathrm{d}A_k} \propto& -\frac{\partial D^R(p_R,p_X)}{\partial A_k}\frac{\partial D^X(p_R,p_X)}{\partial p_X} + \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial A_k}\\
=& -\left\{R_k\left[(\sigma_k-1)+(\epsilon-\sigma_k)\alpha_{Rk}\right]\frac{1}{A_k}\right\}\left\{\sum_{i=1}^{N} X_i\left[(1-\sigma_i)-(\epsilon-\sigma_i)\alpha_{Xi}\right]\frac{1}{p_X}\right\}\\
&+\left\{\sum_{i=1}^{N} R_i\left[1-(\epsilon-\sigma_i)\alpha_{Xi}\right]\frac{1}{p_X}\right\}\left\{X_k(\epsilon-\sigma_k)\alpha_{Rk}\right\}\frac{1}{A_k}\\
=& -(\sigma_k-1)\frac{R_k}{A_k}\frac{1}{p_X}\sum_{i=1}^{N} X_i\left[(1-\sigma_i)-(\epsilon-\sigma_i)\alpha_{Xi}\right]\\
&+(\epsilon-\sigma_k)\alpha_{Rk}\frac{1}{A_k}\frac{1}{p_X}\left[X_k\sum_{i=1}^{N}R_i - R_k\sum_{i=1}^{N}X_i(1-\sigma_i) - \sum_{i=1}^{N}(\epsilon-\sigma_i)\alpha_{Xi}[X_kR_i - R_kX_i]\right].
\end{aligned} \tag{A-4}$$

Factoring $1/p_X$ and multiplying by A_k yields the expression in the main text.

Derivation of equation (7)

Set $\sigma_i = \sigma_j = \sigma$ and $A_i = A_j = A$ in equation (A-4), restore the factored terms, and recognize that, because all sectors are initially symmetrical, $X_i = L/N$ and $R_i = R/N$:

$$\begin{aligned}
A_k \frac{\mathrm{d}R}{\mathrm{d}A_k} =& \frac{\psi R}{p_R}\Bigg\{ -(\sigma-1)R_k \frac{1}{p_X}\sum_{i=1}^{N} X_i\left[(1-\sigma)-(\epsilon-\sigma)\alpha_X\right] \\
&+(\epsilon-\sigma)\alpha_R \frac{1}{p_X}\left[X_k\sum_{i=1}^{N} R_i - R_k\sum_{i=1}^{N} X_i(1-\sigma) - \sum_{i=1}^{N}(\epsilon-\sigma)\alpha_X[X_k R_i - R_k X_i]\right]\Bigg\} \\
&\left\{\frac{\partial D^R(p_R,p_X)}{\partial p_R}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial p_R}\right\}^{-1} \\
=& \frac{\psi R}{p_R}\left\{ -(\sigma-1)\frac{LR}{Np_X}\left[(1-\sigma)-(\epsilon-\sigma)\alpha_X\right] + (\epsilon-\sigma)\alpha_R\frac{LR}{Np_X}\left[1-(1-\sigma)\right]\right\} \\
&\left\{\frac{\partial D^R(p_R,p_X)}{\partial p_R}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial p_R}\right\}^{-1} \\
=& \frac{LR}{Np_X}\frac{\psi R}{p_R}\left\{ -(\sigma-1)\left[(1-\sigma)-(\epsilon-\sigma)\alpha_X\right] + (\epsilon-\sigma)\,\sigma\,\alpha_R\right\} \\
&\left\{\frac{\partial D^R(p_R,p_X)}{\partial p_R}\frac{\partial D^X(p_R,p_X)}{\partial p_X} - \frac{\partial D^R(p_R,p_X)}{\partial p_X}\frac{\partial D^X(p_R,p_X)}{\partial p_R}\right\}^{-1}.
\end{aligned}$$

Using equation (A-3) and canceling terms yields the expression in the main text.

Proof of Proposition 6

Rearranging equation (A-4), we find

$$\begin{aligned}
\frac{\mathrm{d}R}{\mathrm{d}A_k} \propto& \frac{R_k}{p_X}\left[(1-\sigma_k)-\alpha_{Rk}(\epsilon-\sigma_k)\right]\left\{\sum_{i=1}^{N} X_i[(1-\sigma_i)-\alpha_{Xi}(\epsilon-\sigma_i)]\right\} \\
&+(\epsilon-\sigma_k)\alpha_{Rk}\frac{X_k}{p_X}\sum_{i=1}^{N} R_i\left[1-\alpha_{Xi}(\epsilon-\sigma_i)\right]. \qquad \text{(A-5)}
\end{aligned}$$

The term in curly braces on the first line of the right-hand side is negative by the assumption that $\sigma_i > \sigma_i^L$. Using $\alpha_{Rk} + \alpha_{Xk} = 1$, we have that this first line is negative if and only if $\sigma_k \leq \epsilon - \frac{\epsilon-1}{\alpha_{Xk}}$ (which makes the term outside the curly braces positive), where $\epsilon - \frac{\epsilon-1}{\alpha_{Xk}} < 1$. This is compatible with $\sigma_k > \sigma_k^L$ if and only if $\alpha_{Xk} > \alpha_{Rk}$, which holds for $\sigma_k < 1$ if and only if R_k/X_k is sufficiently large.

The last line is negative if $\sigma_i \leq \epsilon - \frac{1}{\alpha_{Xi}}$ for all i, where $\epsilon - \frac{1}{\alpha_{Xi}} < 1$. Note that $\epsilon - \frac{1}{\alpha_{Xk}} \geq \epsilon - \frac{\epsilon-1}{\alpha_{Xk}}$ if and only if $\epsilon \geq 2$. Thus, if $\epsilon > 2$, then $\sigma_i \leq \epsilon - \frac{1}{\alpha_{Xi}}$ for all i and $\sigma_k \leq \epsilon - \frac{\epsilon-1}{\alpha_{Xk}}$ ensure that both lines are negative. If $\epsilon \leq 2$, then $\sigma_i \leq \epsilon - \frac{1}{\alpha_{Xi}}$ for all i ensures that both lines are negative. And note that $\epsilon - \frac{1}{\alpha_{Xi}} > \sigma_i^L$ if and only if $\alpha_{Ri}/\alpha_{Xi} < \epsilon - 1$, which holds if and only if R_i/X_i is sufficiently large.

Proof of Proposition 7

Consider equation (A-5). The term in curly braces on the first line of the right-hand side is negative by the assumption that $\sigma_i > \sigma_i^L$. Using $\alpha_{Rk} + \alpha_{Xk} = 1$, we have that this first line is positive if and only if $\sigma_k \geq \epsilon - \frac{\epsilon-1}{\alpha_{Xk}}$ (which makes the term outside the curly braces positive). The last line is positive if and only if $\sigma_i \geq \epsilon - \frac{1}{\alpha_{Xi}}$ for all i. Thus both lines are positive if $\sigma_i \geq \max\{\epsilon - \frac{\epsilon-1}{\alpha_{Xk}},\ \epsilon - \frac{1}{\alpha_{Xi}}\}$ for all i. And $\epsilon - \frac{\epsilon-1}{\alpha_{Xk}} \geq \epsilon - \frac{1}{\alpha_{Xi}}$ if and only if $\epsilon \leq 2$.

Proof of Proposition 9

Factoring $(\epsilon - \sigma_k)\alpha_{Rk}$ and rearranging, we have that the general equilibrium component in equation (8) is proportional to

$$R_k \sum_{i=1}^{N} X_i \sigma_i + \sum_{i=1}^{N} \left[1 - (\epsilon - \sigma_i)\alpha_{Xi}\right] \left[X_k R_i - R_k X_i\right]. \tag{A-6}$$

The first term is positive. Each summand in the second term is the product of two ambiguous terms. Note that

$$1 - (\epsilon - \sigma_i)\alpha_{Xi} \geq 0 \Leftrightarrow \sigma_i \geq \epsilon - \frac{1}{\alpha_{Xi}}.$$

Also note that

$$X_k R_i - R_k X_i \geq 0 \Leftrightarrow \frac{R_i/X_i}{R_k/X_k} \geq 1.$$

From equations (4) and (5), we have:

$$\frac{R_i}{X_i} = A_i^{\sigma_i - 1} \left(\frac{p_X}{p_R} \frac{1-\kappa}{\kappa}\right)^{\sigma_i},$$

and thus:

$$\frac{R_i/X_i}{R_k/X_k} = \frac{A_i^{\sigma_i - 1}}{A_k^{\sigma_k - 1}} \left(\frac{p_X}{p_R} \frac{1-\kappa}{\kappa}\right)^{\sigma_i - \sigma_k}.$$

Now fix $\sigma_i = \sigma_j = \sigma$ for all i, j and assume that $\sigma_i \geq \epsilon - \frac{1}{\alpha_{Xi}}$ for each i. Then each term in the ambiguous sum in (A-6) is positive if, for all i,

$$\frac{R_i/X_i}{R_k/X_k} \geq 1,$$

which in turn holds if and only if

$$A_k^{1-\sigma} \geq A_i^{1-\sigma}.$$

If $\sigma < 1$, then this condition holds if $A_k \geq A_i$, for all i. If $\sigma > 1$, then this condition holds if $A_k \leq A_i$, for all i.

Now fix $\sigma_i = \sigma_j = \sigma$ for all i, j and assume that $\sigma_i \leq \epsilon - \frac{1}{\alpha_{Xi}}$ for each i. Then each term in the ambiguous sum in (A-6) is positive if, for all i,

$$\frac{R_i/X_i}{R_k/X_k} \leq 1,$$

which in turn holds if and only if

$$A_k^{1-\sigma} \leq A_i^{1-\sigma}.$$

If $\sigma < 1$, then this condition holds if $A_k \leq A_i$, for all i. If $\sigma > 1$, then this condition holds if $A_k \geq A_i$, for all i.